THE PROGRESSIVE WEAR OF TUBES: THE VOLUMES OF THE INTERSECTIONS OF CYLINDERS WITH EACH OTHER AND WITH FLATS

J. WARBURTON and R. BRADFORD

Scientific Services Department, Central Electricity Generating Board, South Western Region, Bedminster Down, Bridgwater Road, Bristol BS13 8AN (U.K.)

(Received January 10, 1986; accepted March 27, 1986)

Summary

It is shown that cylinders can wear against each other or against flats in only a very few ways, each of which can be defined geometrically. For each geometry the volume of any intersection varies with a few easily measured dimensions. When dealing with a wear scar on a tube it is relatively easy to relate the volume already lost as a fraction of the volume that must be removed before any particular depth is reached. The results are presented graphically so that calculations can be reduced to a minimum and the results made as widely applicable as possible.

The methods developed here have already been employed to predict the lifetimes of irreplaceable tubes showing wear scars. Amongst other possible uses of these methods, one discussed in this paper is a better way of calculating specific wear rates in wear testing using crossed cylinder geometries.

1. Introduction

Vast numbers of tubes exist, for example, in heat exchangers, where the transfer of heat is facilitated by turbulence within the fluid media. Turbulence leads to tube motion, often resulting in fretting or impact wear of the tubes whenever they make contact with the following solid bodies within the shell side: other tubes; steadies; brackets; the shell wall etc. as Ko [1] has described. The fluid flow tends to remove loose wear products from the active sites and may allow wear to proceed unabated or until contact is lost.

This paper shows that the vast majority of the multitudinous possible interactions of tubes, or any cylindrical section, with each other or with flat-sided bodies, can be reduced to a few relatively simple cases which can each be defined geometrically in a general way. These general cases can be simply used in many practical applications to show the amount of tube life expired. It is equally possible to use the same expressions to predict the size of wear scars for particular contact conditions; although here essential information

0043-1648/86/$3.50

may often not be available. Although the approach adopted is directly applicable to the wear of tubes, other cylindrical bodies such as bolts and stays are included in the analysis. Throughout this paper wear is defined as the common volume of the intersection of geometric shapes; volume removal rates are taken as being invariant with time. Nowhere are definite statements made about real wear rates: the purpose of the document is to show how geometric considerations allow many problems to be bounded perhaps more rigidly than might be expected.

2. Tube sections with scars having a straight base

The cross-section of a tube is an annulus (Fig. 1). If the tube has a scar with a straight base the depth of the scar H can be determined from the scar width C, perpendicular to the plane of the tube axis, and the tube's external diameter $2r$. The quantities are related thus

$$\frac{C}{2}\frac{C}{2} = H(2r - H)$$

or

$$C = 2\{H(2r - H)\}^{1/2} \tag{1}$$

$$H = \frac{2r \pm (4r^2 - C^2)^{1/2}}{2}$$

the larger root is the remaining ligament case.

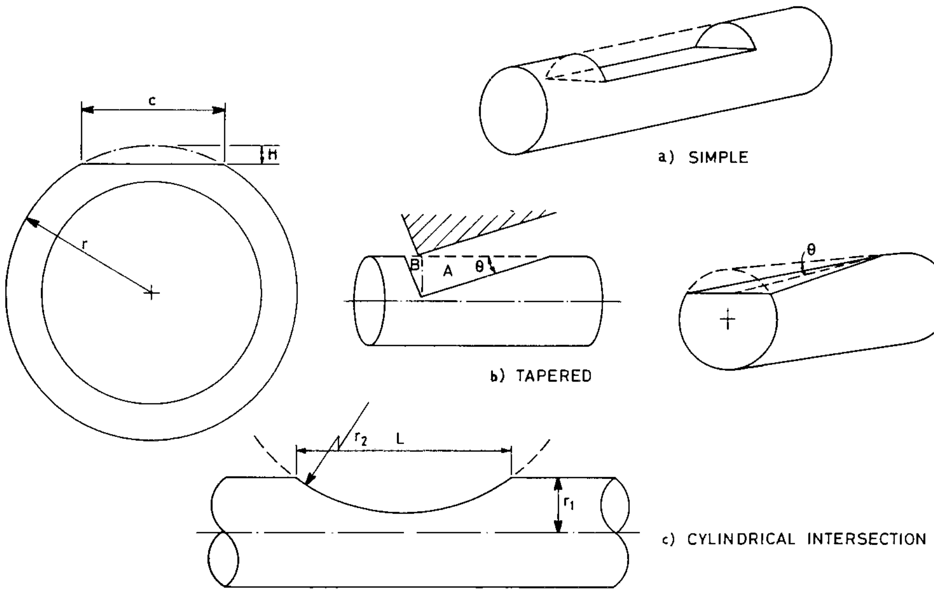

Fig. 1. Flat-based scar types.

Now for any tube there will be a critical scar depth; if the scar becomes deeper the tube will leak. For some cases the unsupported part of the scar base will be relatively insignificant in structural terms and often it may be sufficient to equate the critical scar depth with the tube wall thickness. For very long scars the maximum hoop stress may limit the scar depth. In conditions where creep is important the rate of wear will also affect the determination of a critical scar depth. Since it is normal to know the radius and wall thickness of the tube the critical scar width can, in principle, always be obtained from the relevant material properties. Conservatism may require the critical depth to be defined by the occurrence of significant plastic or creep strains in the ligament causing distortions of the tube cross-section away from the perfect circle assumed. Likewise the manufacturing process may produce oval tubes: this may be countered by regarding the tube as being of smaller radius.

Scar widths are usually much easier to measure than scar depths and the relationship above shows that even very shallow scars are relatively wide. Having a straight baseline does not define the scar in three dimensions. Three separate regular cases should be considered.

The first case is a simple flat-based scar (Fig. 1(a)) whose cross-section is either invariant or bounded by discontinuities. In such a scar the affected tube length does not vary with scar depth. A thin hard straight edge or a flat unwearing rectangle whose face is parallel to the tube axis could produce such a wear pattern; as would the rubbing together of two equally wearing parallel similar tubes.

The second case to be considered is where the cross-section of the scar base in the line of the tube axis is itself part of another circle of any radius (Fig. 1(c)). Such a shape could be produced by the intersection of two cylinders at right angles. The relationship between scar depth and length is determined by a form similar to eqn. (1).

The third case is the tapered scar, one which 'runs out' as in Fig. 1(b). Such a scar could be produced by a wear-resistant rectangular block whose face is angled to the tube axis. In this case there would be an invariant relationship between scar depth and length. Other regular scar geometries exist but are unlikely to result from the interactions of cylinders or of a cylinder and a flat.

2.1. The simple flat-based scar

This, the simplest scar, has a cross-sectional area A defined by the expression

$$A = r^2 \arccos\left(\frac{r-H}{r}\right) - (r-H)\{r^2-(r-H)^2\}^{1/2} \tag{2}$$

The volume removed is the product of the length (a constant) and the area. This enables the relative volumes of scars to be easily determined. Most useful is the ratio of a measured scar area to that of the area at the critical scar depth. A graphical representation of eqn. (2) is given in Fig. 2 where the

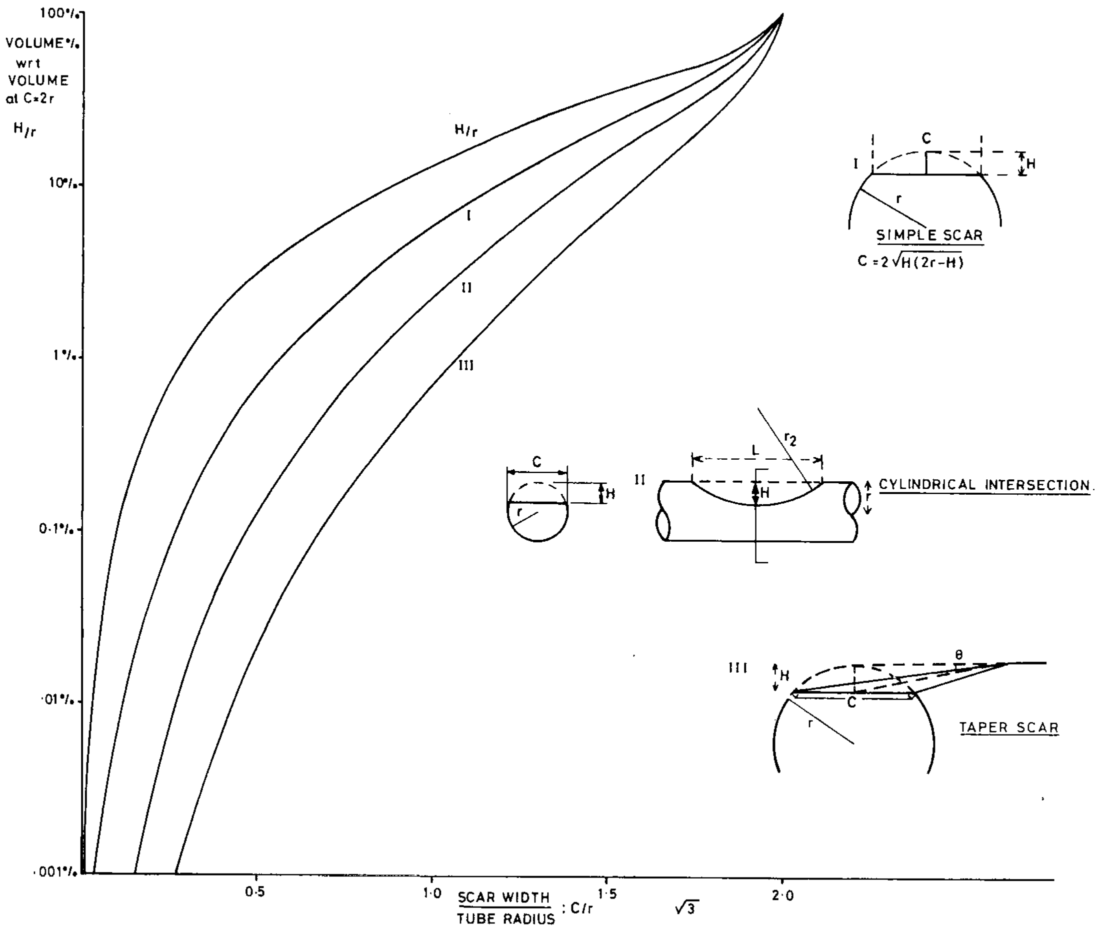

Fig. 2. Scar width *vs.* depth and volume for the three separate geometries.

area of any segment of width C/r is given with respect to the area when the scar width equals the diameter, *i.e.* with respect to $\pi r^2/2$ or r^2 arccos 0. The critical value of H/r gives a corresponding value of A (eqn. (2)) at the same C/r. From a measured scar width the value of A is read from the same line. Both are expressed relative to $\pi r^2/2$, so the ratio of the actual scar area to the critical scar areas can be found simply. If the volumetric wear rate is assumed constant then this ratio is the fractional life expired. An example of this technique is given in Section 5.1.

2.2. *The cylindrical intersection scar*

The intersection of a tube of radius r_1 by another of radius r_2 at right angles is shown in Fig. A1 in Appendix A. The derivation of the volume of intersection V is given in Appendix A. Both a numerical and an approximate integration are given. The latter, eqn. (A8), gives

$$V = \pi H^2 (r_1 r_2)^{1/2} \left(1 - \frac{H}{8r_1} - \frac{H}{8r_2}\right) \tag{3}$$

The errors in this expression are shown in Fig. A3 in Appendix A. Considering the assumptions made, the expression is remarkably accurate for $H < r_1$ and r_2 and the extent of the inaccuracy is very little affected by the relative sizes of r_1 and r_2.

Thus a single line, II on Fig. 2, shows the progress of all such intersections with very little error. This line, intermediate between lines I and III at all points, is the result of the more precise numerical integration of eqn. (A5) in Appendix A.

The volume given in Fig. 2 as 100% is based on the diameter since intersection scars are asymmetric when $H > r_1$ or r_2 and $r_2 < 2r_1$. The geometrical intersection then differs from a wear scar; the volume, when $r_1 = r_2 = H$, is $2.323 r^3$.

The same line in Fig. 2 is of more use than just relating the relative wear of crossings at 90°. Appendix A shows that it also applies to straight tubes crossing at any angle θ for which

$$V \approx \pi H^2 \operatorname{cosec} \theta (r_1 r_2)^{1/2} \left(1 - \frac{H}{8r_1} - \frac{H}{8r_2}\right) \tag{4}$$

As cosec θ is a constant, the line II in Fig. 2 also represents the progress of wear through tubes which cross each other at any angle. The exact measurement of C can become awkward if θ approaches zero as Fig. A4 shows. The same line can also be used for the intersection of bent tubes, although the exact relationship looks rather different (Appendix A).

2.3. *The tapered base scar*

The cross-sectional area of such a scar at any point has the form of eqn. (2) but the depth varies along the scar length. If X is the scar axial length from a depth of zero to H, the relationship X/H is constant and equal to the cotangent of the scar angle θ. Appendix B shows how the expression for the volume of the scar can be determined

$$V = \cot\theta\left(r^2\left[\{r^2-(r-H)^2\}^{1/2}-(r-H)\arccos\frac{r-H}{r}\right]\right.$$
$$\left.-\frac{r^3}{3}\left\{1-\left(\frac{r-H}{r}\right)^2\right\}^{3/2}\right) \tag{5}$$

for the part 'A' of Fig. 1(c). Part 'B' has a similar form but instead of cot θ has, for an unwearing rectangular counterface, $\cot(90-\theta)$ or $\tan\theta$ by the term in the major parentheses.

If the scar volume at any depth, as determined from the scar width, is compared with the volume of a scar whose width is equal to $2r$

$$V = \tfrac{2}{3} r^3(\cot\theta + \tan\theta)$$

then the line III on Fig. 2 results.

For a given volume of material removed in a fixed scar length the tapered scar shows the greatest maximum depth of penetration, the simple flat-based scar the least. However, the tapered scar may well have a greater critical depth as the minimum ligament is localized.

3. Application of the wear geometries

The three general eqns. (2), (4) and (5) between them cover the types of interactions likely to be found in practice but the application of the lines in Fig. 2 needs some care. The factors which should be considered are that tubes are not necessarily straight and that the bases of scars are not always flat.

3.1. Bent tubes

Figure 3 shows a tube of radius r_1 intersected by another of radius r_3 bent around a radius r_2. The volume of the intersection at any depth H has already been expressed in terms of r_1 and r_2 if the angle θ between the tube axes in plan is zero. For H small compared with r_2 the volume at a given depth is proportional to $(r_2)^{1/2}$. Hence line II in Fig. 2 is robust in that the progression of wear volumes in two other cases can be plotted by its use. Firstly, it does not require r_2 to be known; secondly, it may be used when r_2 is not constant. This latter case is of use when θ is not zero, so that the intersection is a chord of an ellipse, or for vibrating or thermally bent tubes whose bends are not strictly parts of any circle.

Obviously, the tube of radius r_1 may also be bent but the value of r_2 can be calculated from the difference in the reciprocals of the radii of the contacting bends. If r_2 is needed a derivation based on scar length is given in Section 3.2.2.

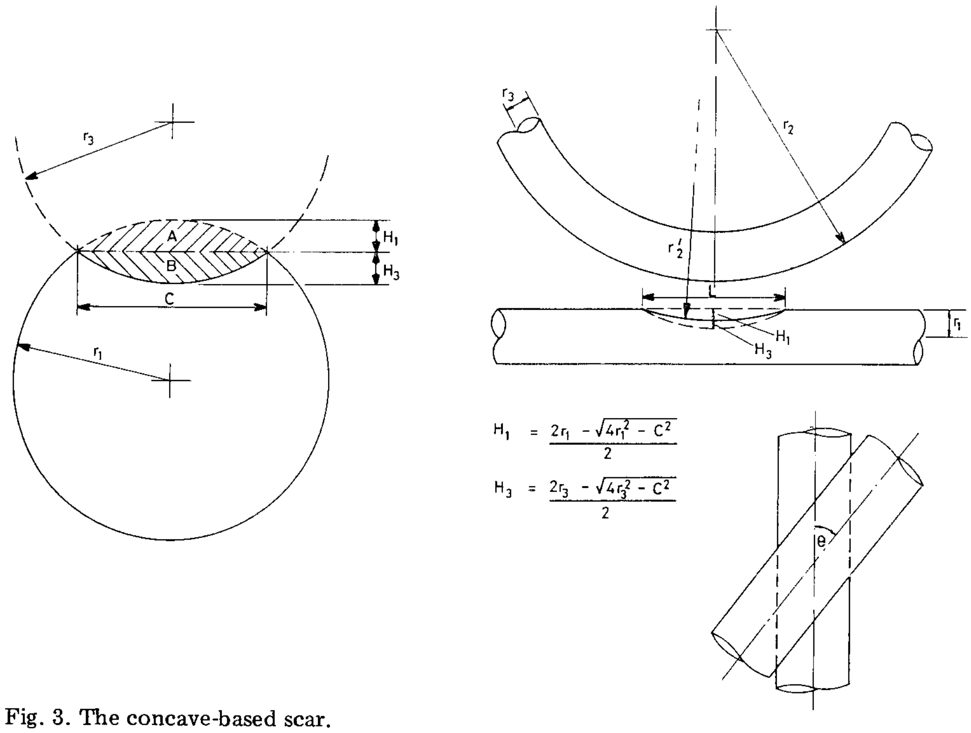

Fig. 3. The concave-based scar.

3.2. Scars with concave bases

In many instances the bases of scars will not be flat. If two parallel tubes rub the equality of wear rates needed to produce truly flat scars must be fortuitous. One tube will almost certainly wear less than the other and the extreme case is observed when one tube shows no wear. In such a case Fig. 3 shows that the base of the scar is concave with a radius equal to that of the wear-resistant tube and hence for tubes of equal radius the depth is exactly twice that which would be anticipated from the previous analysis.

3.2.1. Concave-based simple scars

The analysis of such scars is still simple, it is only necessary to replace the critical value of H/r with one which is one half the size and from this read off the associated values of C/r and the equivalent percentage volume. Although nominally similar tubes should have nominally similar wear rates we can be confident of scars being no worse than this analysis shows. If the unwearing tube has a smaller radius the resulting scar will be deeper but, nevertheless, the maximum scar depth can always be determined from the tube radii and the scar width.

3.2.2. Concave-based intersection scars

Similar reasoning enables concave-based intersections to be considered since bent tubes may cause concave-based scars on straight tubes. Figure 3 shows the intersection of a straight tube of radius r_1 by a tube of radius r_3 which is itself bent in a radius of r_2. From C the depths of each part of the scar, A and B, can be obtained. If each part of the scar has depth H_1 and H_3 respectively (from Fig. 3) the total depth of the intersection, $H = H_1 + H_3$, is given from the scar length L and r_2 thus

$$r_2 = \frac{L^2}{8(H_1 + H_3)} + \frac{H_1 + H_3}{2} \tag{6}$$

when $r_1 = r_3$, as will often be the case, $H_1 = H_3$

$$\frac{r_2}{r_1} = \frac{L^2 + 16H^2}{2C^2 + 8H^2}$$

or (7)

$$\frac{r_2}{r_1} \approx \frac{L^2}{2C^2}$$

Although the area of part A is independent of r_3 and the area of part B is independent of r_1, the volumes of each part are interdependent as Appendix A shows. Although the volume is in two separate parts, to a close approximation it can be regarded as having the same form as line II in Fig. 2.

It is difficult to envisage a practical situation in which two cylinders can give a concave-based taper scar since it requires the extrapolated axes to pass through each other. A kinked tube wearing against a straight one might produce such an effect. Nevertheless, the application of the principle remains identical.

3.3. Convex-based scars

Straight-based scars may be caused by the interactions of cylinders which wear against plane surfaces which do not. Practically, wear will occur on both surfaces giving the cylinder a convex scar base. Figure 4 shows the result of the interaction of an equally wearing cylinder and flat. The radius of the base of the scar is shown to be $2r - 3H/4$. It is doubtful whether real scars could be measured to the accuracy necessary to observe this relationship. None the less, it can be designated as the 'most likely' case for the interaction of a cylinder and flat; the straight base is the worst case. In the convex-based scars the depth is always less than that expected from eqn. (1).

If a tube is known to be wearing against a flat, and both have similar wear properties, then the most likely scar will have a depth one half of that given in the analysis leading to eqn. (1) and the evaluation of an individual scar proceeds by assuming a critical value of H/r twice the real value.

4. Application to wear rate testing by crossed cylinder methods

Irrespective of the geometry of the wearing pair, the distribution of wear cannot be measured without taking the pair apart. Nevertheless, the

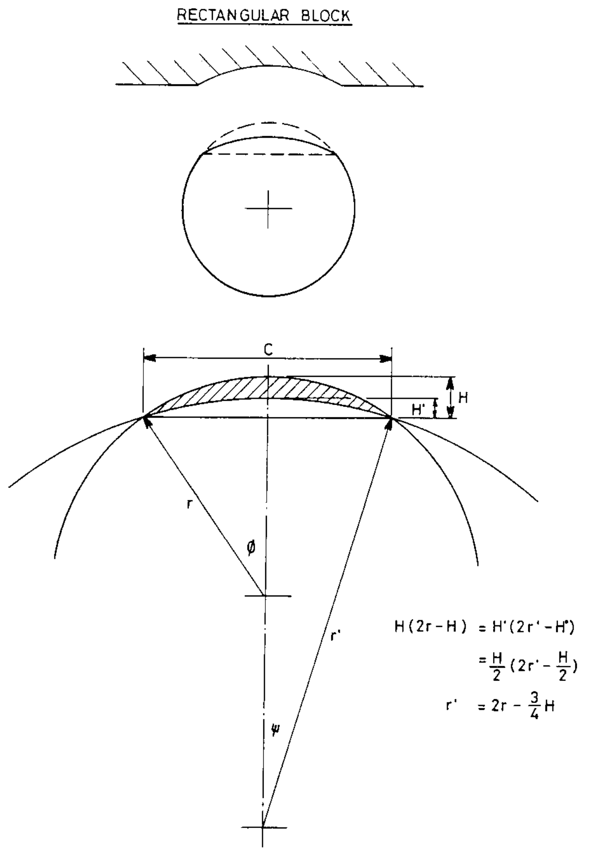

Fig. 4. The convex-based scar (▨, area worn away).

total wear can be determined from the change in height of two wearing objects of known geometry. For equally wearing crossed cylinders of equal radius the saddle-shaped wear scar of indeterminate radius has a volume half that of the simple intersection. This saddle is the rotation of the flat-based simple scar through 90°, just as rotation through 90° of the concave-based parallel tube scar gives the flat-bottomed intersection scar.

In wear tests it is not uncommon to study the fretting of crossed cylinders since alignment problems are eliminated. Wear is often determined by very accurate weighing or careful multiple profilometer traces which provide a pseudo three-dimensional image. Since $V \propto H^2$, the mass loss $V\rho$ (where ρ is the density) is of second order in the small quantity H and so it is better to measure H directly. The expression derived in Appendix A and an example here demonstrate the sensitivity of the method.

If two crossed cylinders of 5 mm radius are discovered to have fretted so that their total height has reduced by 10 μm, the volume removed can be determined most easily from the expression given in Appendix A

$$V = \pi(0.01)^2(5 \times 5)^{1/2}\left(1 - \frac{0.01}{8 \times 5} - \frac{0.01}{8 \times 5}\right) = 15.7 \times 10^{-4}\ \text{mm}^3$$

if $\rho = 8$ mg mm^{-3} this is equivalent to a mass loss on the combined cylinders of 12 μg. Such a mass loss may probably not be accurately determined from weighing and might be difficult to measure accurately from profilometer traces. This technique presumes that wear is the volume of material displaced. The corresponding scar width is 0.63 mm or more if debris were retained in the contact. Transferred material can affect both the scar widths and the changes in height. However, the use of the geometric relationships is probably no less accurate than profilometry and rather simpler. If the two cylinders are of different materials the individual wear rates cannot be obtained for each quite so easily.

5. Examples of wear estimation based on scar width

5.1. The evaluation of straight-based scars

Since three of the four lines on Fig. 2 purport to cover all the wear volumes of scars with straight-based sections orthogonal to the axis of the cylinder it is useful to refer to the graph for a practical example. Suppose a tube of 50 mm radius has a critical scar depth of 10 mm. Then the critical value of $H/r = 20\%$, at which depth $C = 1.2r$. At this point the relative volumes, as given by the lines I, II and III, are 10.5%, 5.2% and 1.95% respectively. If a scar is noted to be 25 mm wide, *i.e.* $C = 0.5r$, then the depth of the scar is 3.2% of the radius or 16% of the critical depth. The volumes removed are 0.7%, 0.125% and 0.02% respectively of the diametral scars; equivalent to 6.6%, 2.4% and 1% of the volumes to breakthrough in each case. This information results solely from measuring the scar width and deciding on the geometry of the interaction. This in itself is relatively straightforward since type I scars have constant widths along their lengths or are produced by actions of thin edges against tubes. Type II scars will be widest in the middle and run out either end. Type III scars will normally be obviously at their widest very close to one end.

As already noted, limits can be put on the severity of scars dependent upon their causes. Thus the figures shown above are the worst that could result from the interaction of a flat with the tube (types I and III) or another tube (types I and II). However, the straightforward use of Fig. 2 implies that the volumetric wear rate remains constant. It is quite conceivable that two tubes will wear out of contact before penetration: in that case the wear has a finite limit. It is also possible that the wear rate will not be constant, even if all the parameters governing it remain steady, as Aldham *et al.* [2], amongst others, demonstrate. Generally, wear rates and contact loads, if they vary at all, decrease as wear progresses, so the assumption of volumetrically constant wear rates will provide a built-in pessimism. Wear can also affect the dynamic response of the tubes and so change the wear rates. Such a change is not easy to predict but in many cases the dimensional changes which take place before the tube fails will be quite minor.

It is instructive to note that quite wide scars can be very shallow (see Fig. 2): a scar of width 0.3 of the radius is but 1% of the radius deep and the proportionate volumes associated with type I scars are an order of magnitude down on this, type II an order of magnitude less than type I and type III similarly less than type II. Yet such a scar would be 15 mm wide in the example and hence be clearly visible.

5.2. *Curved base scars*

As mentioned in Sections 3.2 and 3.3, the shape of a scar base is likely to differ from a flat depending on how the scar was formed.

If two similar tubes are wearing through each other then in theory the scar could be twice as deep on one tube as the width would imply. In the example used above the analysis proceeds simply by putting the critical value of H/r at 10%, *i.e.* $C/r = 0.88$. Line I is then at 4%. The simple 25 mm scar has thus consumed 0.7/0.04 or 17.5% of the tube life and so on for the other scar types. This then demonstrates the upper bound of such a scar although the flat-based scar is the most likely case for two tubes clashing.

The most likely result of a flat wearing against a tube is the convex-based scar. If equal wear rates can be assumed the critical value of H/r in this case can be put at 40% when $C = 1.6r$. The simple scar has then 29% of the volume of the diametral scar and at $C/r = 0.5$, 0.7% of that volume. The most likely value then becomes 0.7/0.29 or 2.4%. The other scar types are dealt with similarly.

5.3. *Truncated taper scars*

Many taper scars may be of low angle owing to inaccuracies in manufacture, *e.g.* a bracket face notionally parallel to the tube axis. This will give scar lengths relatively enormous compared with the depth but the taper cannot continue unabated; before it wears through, the whole of the bracket width will be wearing into the tube and a truncated taper will result. If the bracket width; if 16% of the critical depth has been penetrated in 50% of the width, then the scar will truncate when it becomes more than 32% of the critical depth. At breakthrough the volume removed can be represented as equivalent to a whole taper, less that of the unworn piece 68% of the depth. Looking at Fig. 2, for H/r critical at 20%, the volume is equivalent to 1.95% of the diametral volume. The volume of the "tail" of the taper, from 68% of H/r Fig. 2, for H/r critical at 20%, the volume is equivalent to 1.95% of the diametral volume. The volume of the "tail" of the taper, from 68% of H/r critical to zero depth, is 0.75% of the diametral volume. Hence the volume of the truncated taper is 1.2%. In the original example 0.02% has been removed; only one sixtieth of the volume to breakthrough.

6. Wear scar predictions not based on scar width

The analysis so far has relied upon measuring scar widths, assuming a likely wear geometry and obtaining the relative amount of wear that has taken place with respect to the amount of wear needed to cause tube failure. By this means the assumption of a volumetrically constant wear rate can be used to predict service life. Another approach is also possible; if wear tests and a knowledge of the likely conditions can be used to predict the volume of wear that will take place in service, the expressions derived can be invoked to show whether penetration will take place. Some simple examples follow which illustrate possible uses of the relationships.

6.1. Example 1

Consider the 50 mm radius 10 mm tube referred to previously. Predictions based on wear rates and energy inputs suggest that it may lose as much as 10^4 mm^3 in volume by wearing against a rectangular bracket angled at 3° to the axis of the tube. How deep and wide will be the scar?

When $H = r$ the volume $V = \frac{2}{3} r^3(\cot\theta + \tan\theta)$, at 3°

$$\cot\theta + \tan\theta \approx 20$$

$$\frac{V}{V_{(H=r)}} = \frac{3 \times 10^4}{2 \times 50^3 \times 20} = 6 \times 10^{-3} = 0.6\%$$

from Fig. 2

$$\frac{C}{r} = 0.97, \quad \therefore C = 48.5 \text{ mm}$$

$$\frac{H}{r} = 12\%, \quad \therefore H = 6 \text{ mm}$$

6.2. Example 2

The tube passes close to a wear-resistant division wall which is very long. If we expect that the tube may lose 10^5 mm^3 owing to wear, how accurately must the tube run be constructed to prevent breakthrough? From Fig. 2

$$H = 0.2r$$

$$V = 0.0195 \times \tfrac{2}{3} r^3 \cot\theta$$

$$\therefore \cot\theta = \frac{10^5 \times 3}{50^3 \times 0.0195 \times 2} = 61.54$$

i.e. $\theta \approx 1°$, so a run which is accurate to 1° will suffice.

6.3. Example 3

In a heat exchanger a tube is bent against the effectively flat wall (Fig. 5). If the tube radius is 10 mm, the wall thickness 2 mm, the tube run

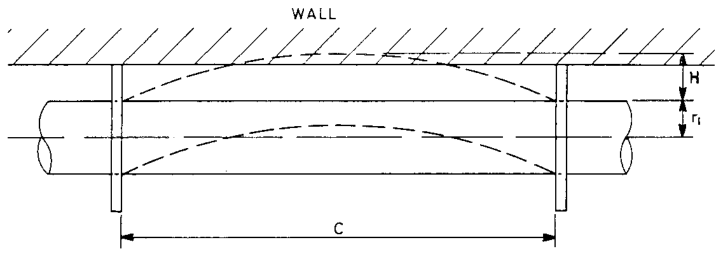

Fig. 5. Example 3.

10^3 mm and the wall at 20 mm from the tube axis, how much material is removed from the tube at breakthrough?

It is best to visualize the shape of the intersection by imagining the shell wall wears and not the tube and considering that breakthrough occurs when the intersection becomes 2 mm deep.

At breakthrough, the deflection of the tube axis will be 12 mm. Hence the radius of the bend will be

$$r = \frac{C^2}{8H} + \frac{H}{2} = \frac{10^6}{8 \times 12} + 6 = 10\,420 \text{ mm}$$

Since the wall of the heat exchanger is effectively flat $r_1 = \infty$ and so the volume of the intersection is given by eqn. (A12) in Appendix A. In this case $r_2 = r_2{}^* = 10\,420$ mm and $r_3 = 10$ mm. The wear volume is then

$$V = \int_0^2 \pi\{h^2(2r_3 - h)(2r_2{}^* - h)\}^{1/2}\, \mathrm{d}h$$

which gives $V = 3918.5$ mm^3.

Although eqn. (3) is not strictly applicable, the value of the expression with the same values of $r_2{}^*, r_3$ and $H = 2$ gives 3953 mm^3, less than 1% different.

6.4. Example 4

Through the centres of two faces of a cube are drilled holes of radius r so they pass through each other. What is the common volume of the intersection?

The volume of the intersection can be represented in the following two equivalent ways: one is as shown in Appendix A; the other can be best visualized as the intersection of four equal radii rods into a cross. To make them fit exactly, each rod would have removed from it two tapered cuts each at 45° to the surface (see Fig. 6).

The volume of any such taper is, from eqn. (5), putting $H = r$

$$V = \cot\frac{\pi}{4}\left(r^2\left[\{r^2 - (r-r)^2\}^{1/2} - (r-r)\arccos\left(\frac{r-r}{r}\right)\right] - \frac{r^3}{3}\left\{1 - \left(\frac{r-r}{r}\right)^2\right\}^{3/2}\right)$$

$$= \tfrac{2}{3}r^3$$

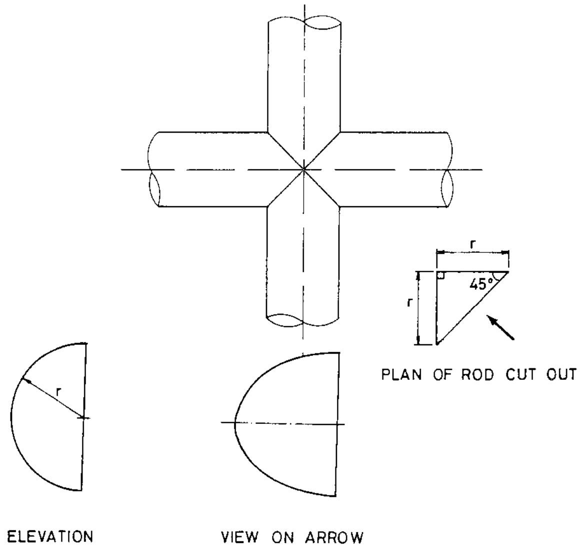

Fig. 6. Example 4.

As there are eight such tapered pieces the volume of the intersection equals $\frac{16}{3}r^3$.

This value may be used to check the accuracy of any numerical integration of eqn. (A5) in Appendix A.

6.5. Example 5

Two crossed cylinders of the same material, each of radius 5 mm, are fretted under a load of 100 N for 10^6 cycles at 10 μm overall stroke. The height of the cylinders falls by 25 μm. What is the wear rate of the material?

From eqn. (3) the volume of the intersection is

$$V = \pi 0.025^2 (5 \times 5)^{1/2} \left(1 - \frac{25}{1000 \times 4 \times 5}\right)$$

$$= 9.8 \times 10^{-3}\ \text{mm}^3$$

$$= 9.8 \times 10^{-12}\ \text{m}^3$$

10^6 cycles at 10 μm $\equiv$ 20 m sliding distance.

$$\therefore \text{ wear rate of material} = \frac{1}{2} \times \frac{9.8 \times 10^{-12}}{100 \times 20}$$

$$\equiv 2.5 \times 10^{-15}\ \text{m}^3\ \text{N}^{-1}\ \text{m}^{-1}$$

7. Inappropriate cases

Implicit in all the foregoing analyses is the assumption that wear, and hence relative movement, proceed in a predictable way until the critical value of H is reached. This requires that the body which causes tube wear is of sufficient dimensions to accommodate a full chord. Then the actual direction of the force is irrelevant since the only direction which would cause the analyses to be undermined is when the force is parallel to the base of the chord; a case in which wear has a finite limit. If the cross-section of the scar is not a full chord, with or without a curved base, gouging arises and then the direction of the force becomes important. As such a direction could probably not be measured easily, and certainly cannot be determined from a single scar measurement, there is no general solution.

8. Conclusions

Scars on cylinders due to other cylinders or flats can be divided into the following three main types: simple; taper; intersection. Each type shows a different relationship of volume to width or depth: with steady wear rates simple scars progress most rapidly in terms of depth, taper scars most slowly. Conversely, for a fixed length and volume removed, taper scars are the deepest and simple scars the shallowest. Within these types it is possible that the base can be flat, concave or convex. Choice of scar type depends upon the assumptions made about the wearing geometry. None the less, it only requires the four lines in Fig. 2 to analyse scars of all types from a knowledge of the tube diameter, wall thickness and scar width. It is also possible to determine the maximum value of the wear by the simple measurement of the change in height of the wearing pair. The relationships can also be employed to predict scar dimensions from estimated total wear volumes assuming any particular geometry.

TABLE 1

Application of base types

Main scar type	*Flat*	*Concave*	*Convex*
Simple	Unwearing flat parallel to tube axis Parallel equally wearing tubes Thin sawing actions	Parallel tubes, one wear resistant	Wearing flat parallel to tube axis
Taper	Unwearing non-parallel flat on tube		Non-parallel wearing flat on wearing tube
Intersection	Crossed cylinders, one wear resistant Bent tube on straight, equally wearing Bent tube sweeping across straight tube axis	Unwearing bent tube on wearing straight	Two wearing crossed cylinders

The uses of the various relationships derived are wide ranging. The intersection case, for example, has use not only for cylinders crossed at 90°, provided material transfer is negligible, but also for bent tubes at any angle or for crossed straight tubes at any angle. Use of the various relationships enables some quite unexpected possible cases to be analysed (see Table 1).

It is comparatively simple to decide which geometry will prevail and which type of base is therefore likely. Recourse to Fig. 2 will enable the significance of any scar to be determined by a suitable adjustment of the critical value of H/r. There may be in-built pessimisms due to the assumption of volumetrically constant wear rates and contact conditions. It has been found most convenient when dealing with scars on many similar tubes to calculate the critical value of H, to find the corresponding value of C and then to plot graphically the percentage of a tube life expired with respect to the scar width for each case. Such a graph then provides a rapid means of analysing wear scars.

Acknowledgment

The authors wish to thank the Director of the South Western Region of the Central Electricity Generating Board for permission to publish this work.

References

1 P. L. Ko, *Wear, 55* (1979) 369 - 384.
2 D. Aldham, R. E. Pendlebury and J. Warburton, *Wear, 106* (1986) 177 - 201.

Appendix A: Tube intersection

A.1. Tubes crossing at right angles

Consider the wearing of a tube of radius r_1 by a tube of radius r_2 whose axes are perpendicular. From Fig. A1 the following relationships can be derived at any depth h:

$$h = H - r_2(1 - \cos\theta) \tag{A1}$$

$$h = r_1(1 - \cos\alpha) \tag{A2}$$

$$Z = r_2 \sin\theta \tag{A3}$$

$$A = r_1^2(\alpha - \tfrac{1}{2}\sin 2\alpha) \tag{A4}$$

$$V = \int A \,\mathrm{d}z$$

$$= 2r_1^2 r_2 \int_0^{\arccos(1-H/r_2)} \left(\alpha - \frac{1}{2}\sin 2\alpha\right) \cos\theta \,\mathrm{d}\theta \tag{A5}$$

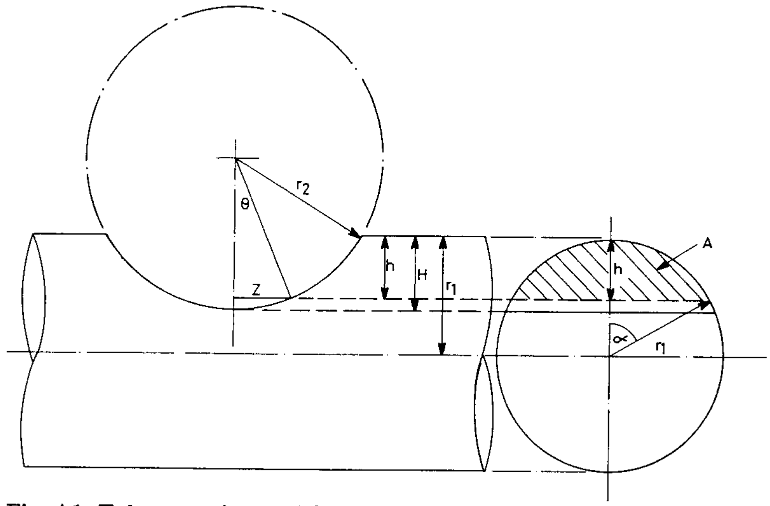

Fig. A1. Tubes crossing at right angles.

(from eqn. (A3)) where, from eqns. (A1) and (A2)

$$\alpha = \arccos\left\{1 - \frac{H}{r_1} + \frac{r_2}{r_1}(1 - \cos\theta)\right\}$$

This exact expression was used for the numerical integration which gave rise to line II in Fig. 2 with $r_1 = r_2$.

In plan, the intersection of two cylinders at right angles has an elliptical outline but rectangular contours at any height h (Fig. A2). If the maximum scar width is C and the width at any depth h is c, the maximum length C' and at depth h, c', then the area of the rectangle is cc'.

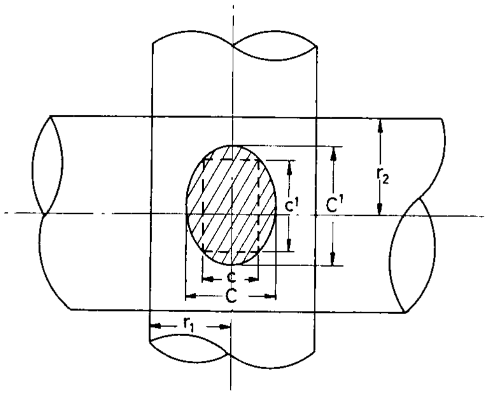

Fig. A2. Tubes crossing at right angles (plan view).

Hence the volume of the intersection

$$V = \int_0^H cc' \, \mathrm{d}h$$

$$= 4\int_0^H [(2hr_1 - h^2)\{2r_2(H-h) - (H-h)^2\}]^{1/2} \, \mathrm{d}h \qquad \text{(A6)}$$

The first order term in H gives

$$V \approx 8\int_0^H \{r_1 r_2 h(H-h)\}^{1/2} \, \mathrm{d}h$$

$$\approx 8(r_1 r_2)^{1/2}\left[\left(\frac{h - \dfrac{H}{2}}{2}\right)\{h(H-h)\}^{1/2} + \frac{H^2}{8} \arcsin\left(\frac{h - \dfrac{H}{2}}{\dfrac{H}{2}}\right)\right]_0^H$$

$$\approx \pi H^2 (r_1 r_2)^{1/2} \qquad \text{(A7)}$$

Expanding to give the second order term in H

$$V = 8(r_1 r_2)^{1/2} \int_0^H \{h(H-h)\}^{1/2}\left(1 - \frac{h}{4r_1} - \frac{H-h}{4r_2}\right) \mathrm{d}h$$

If the mean value of h within the major parentheses is taken as $H/2$ then the correcting term may be used as a constant to give

$$V \approx \pi H^2 (r_1 r_2)^{1/2}\left(1 - \frac{H}{8r_1} - \frac{H}{8r_2}\right) \qquad \text{(A8)}$$

A similar expression may be obtained by means of an approximate integration of eqn. (A5). The difference between this approximate integration and the numerical integration is shown in Fig. A3.

A.2. Cylinders crossed at any angle

When the cylinders cross at an angle θ, and have a depth H of intersection, the width C is unaffected as measured orthogonal to the tube axis even though, as Fig. A4 shows, no part of the scar has the unbroken width C in that direction. Similarly, the value of C' orthogonal to the axis of the other tube is unaffected. The area of the resulting parallelogram contour is cc' cosec θ. As cosec θ is constant throughout all the contours the volume of the tube crossing scar is

$$V \approx \pi H^2 \operatorname{cosec} \theta \, (r_1 r_2)^{1/2}\left(1 - \frac{H}{8r_1} - \frac{H}{8r_2}\right) \qquad \text{(A9)}$$

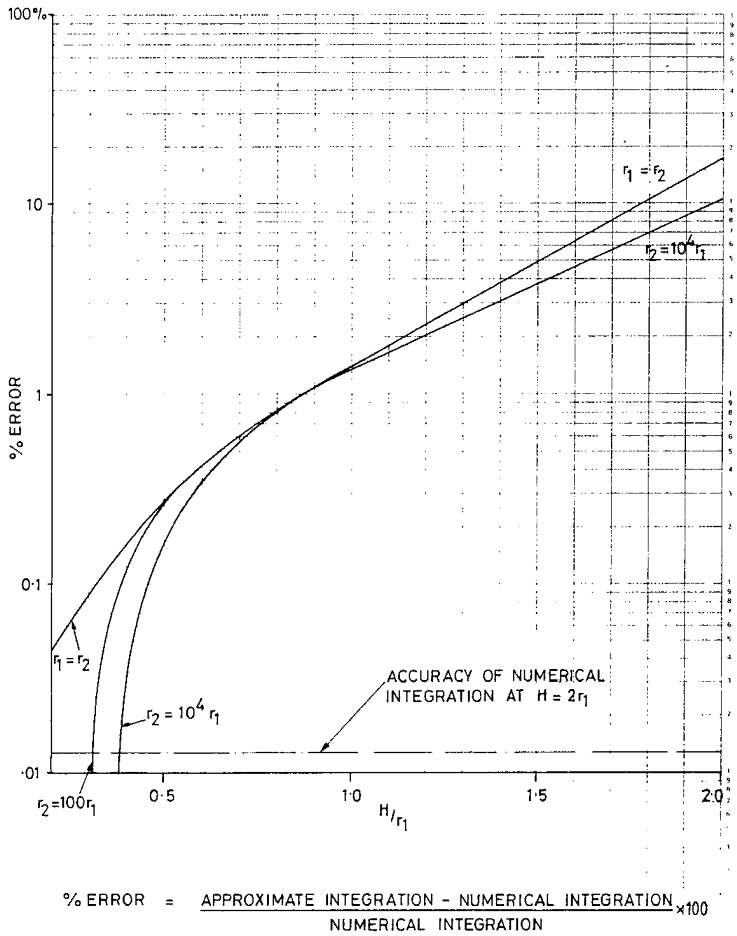

Fig. A3. Errors of approximate integration.

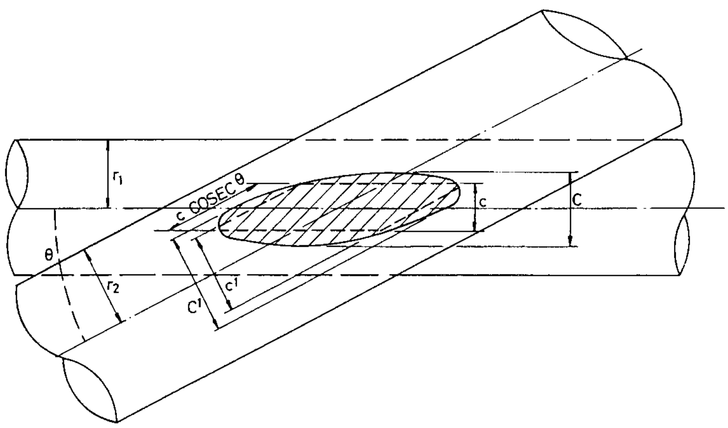

Fig. A4. Cylinders crossing at an angle θ.

The validity of this approach was checked by the numerical integration of the cross-section of the scar along the tube axis and it yielded indistinguishable results.

However, the above diagram shows that the scar width C may become difficult to measure accurately when θ is small. If θ is nominally zero, obviously the volume tends to infinity and the progression of the intersection is then shown by line I in Fig. 2.

A.3. Bent cylinder crossing a straight cylinder

If a straight tube of radius r_1 is cut by a tube of radius r_3 bent in a curve of radius r_2, as shown in Fig. 3, and the bent pipe does not wear, then the volume of intersection can be regarded as in two parts, A and B. The volume of part A is that of a cylinder intersected by another of radius $r_2{}^1$ and it is simple to show that

$$r_2{}^1 = \left(r_2 - \frac{H_3}{2}\right)\left(1 + \frac{H_3}{H_1}\right) - \frac{H_3}{2} \tag{A10}$$

Unless either $r_1 = \infty$ or $r_3 = r_1$ the relative values of H_1 and H_3 depend upon C. However, the volume of part A is given with sufficient accuracy by V_A

$$V_A = \pi H_1{}^2 (r_1 r_2{}^1)^{1/2}\left(1 - \frac{H_1}{8r_1} - \frac{H_1}{8r_2{}^1}\right)$$

The volume B is in plan, elliptical in both outline and contour. Being bounded by the curves $r_2{}^1$ and r_2, it has an effective radius in the plane of axis of the straight tube of $r_2{}^*$ where

$$\frac{1}{r_2{}^*} = \frac{1}{r_2} - \frac{1}{r_2{}^1} \tag{A11}$$

For $r_1 = r_3$ and $h < r_1$, then $r_2{}^1 = r_2{}^* \approx 2r_2$.

The volume B is given by

$$V_B = \int_0^{H_3} \pi\{h^2(2r_3 - h)(2r_2{}^* - h)\}^{1/2}\, dh \tag{A12}$$

which, when $r_2{}^*$ is a constant, as it will be for $r_1 = \infty$ or $r_2/H_3 \to \infty$, is a standard integral. Given the substitution $X = 4r_3r_2{}^* - 2(r_3 + r_2)h + h^2$ this is of the form

$$\frac{V_B}{\pi} = \left[\frac{X(X)^{1/2}}{3} - \frac{(r_3 + r_2{}^*)}{2}(h - r_3 + r_2{}^*)(X)^{1/2} + \frac{(r_3 + r_2)(r_3 - r_2{}^*)}{2}\log_e\{2(X)^{1/2} + 2h - 2(r_3 + r_2{}^*)\}\right]\Bigg|_0^{H_3} \tag{A13}$$

The last term of this unwieldy function may also be written as a hyperbolic function. Crucially, compared with the volume at $H_3 = r_3$, the expression for V_B progresses with C less rapidly than V_A. The interaction of r_1 (via H_1), r_2 and r_3 precludes a general graphical representation for all r_1, r_2 and r_3 and, as $r_2{}^*$ is not usually constant, the expression is not generally integrable. The use of the expression

$$V_B = \pi H_3{}^2(r_3 r_2{}^*)^{1/2}\left(1 - \frac{H_3}{8r_2{}^*} - \frac{H_3}{8r_3}\right)$$

will be pessimistic. The difference between this type and all others considered is that the part B has no independent existence unless $r_1 = \infty$ and normally appears, when there is an unwearing tube, as only a part of the wear volume: for the more normal cases, for example equally wearing equal radii tubes, part B does not exist.

Appendix B: The simple taper scar

Let X be the scar length, x any point along the scar, θ the scar angle and $h = H(1 - x/X)$ the depth at x (Fig. B1).

The area of the segment at point x

$$A(x) = r^2 \arccos\left(\frac{r-h}{r}\right) - (r-h)\{r^2 - (r-h)^2\}^{1/2} \tag{B1}$$

If V equals the volume of cut-out

$$\mathrm{d}V = A(x)\ \mathrm{d}x = A\frac{\mathrm{d}x}{\mathrm{d}h}\ \mathrm{d}h$$

$$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{X}{H}$$

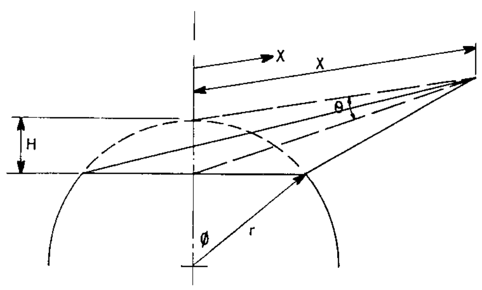

Fig. B1. The simple taper scar.

$$\mathrm{d}V = -\frac{X}{H} A \,\mathrm{d}h$$

$$V = -\frac{X}{H} \int_{x=0}^{X} A \,\mathrm{d}h = \frac{X}{H} \int_{h=0}^{H} A(h) \,\mathrm{d}h$$

For the first part of eqn. (B1) above substitute for $\eta = 1 - h/r$

$$\int_0^H \arccos\left(\frac{r-h}{r}\right) \mathrm{d}h = -r \int_{\eta=1}^{\frac{r-H}{r}} \arccos \eta \,\mathrm{d}\eta = -r\{\arccos \eta - (1-\eta^2)^{1/2}\}\Big|_1^{\frac{r-H}{r}}$$

$$= \{r^2 - (r-H)^2\}^{1/2} - (r-H) \arccos\left(\frac{r-H}{r}\right)$$

For the second part of eqn. (B1) above

$$\frac{r-h}{r} = \cos \phi$$

$$\therefore r - h = r \cos \phi$$

$$\int_0^H (r-h)\{r^2 - (r-h)^2\}^{1/2} \,\mathrm{d}h = \int_{\phi=0}^{\arccos\left(\frac{r-H}{r}\right)} r \cos \phi \; r \sin \phi \; r \sin \phi \,\mathrm{d}\phi$$

$$= r^3 \int_{\sin\phi=0}^{\left\{1-\left(\frac{r-H}{r}\right)^2\right\}^{1/2}} \sin^2 \phi \,\mathrm{d}(\sin \phi)$$

$$= \frac{r^3}{3}\left\{1 - \left(\frac{r-H}{r}\right)^2\right\}^{3/2}$$

combining the two parts

$$V = \frac{X}{H}\left(r^2\left[\{r^2 - (r-H)^2\}^{1/2} - (r-H) \arccos\left(\frac{r-H}{r}\right)\right] - \frac{r^3}{3}\left\{1 - \left(\frac{r-H}{r}\right)^2\right\}^{3/2}\right) \tag{B2}$$

where

$$\frac{X}{H} = \cot \theta$$