Collapse Solution for a Thin Cylinder under Internal Pressure (Ends Capped) and Bending Moment Assuming the Tresca Criterion

Done quickly, so no guarantees…

For Mises the lower bound solution is derived in http://rickbradford.co.uk/BoundTheorems.pdf and the result is circular interaction,

Mises:
$$\left(\frac{\sqrt{3}}{2}\frac{P}{P_0}\right)^2 + \left(\frac{M}{M_0}\right)^2 = 1$$

where,
$$P_0 = \frac{\sigma_y t}{R} \quad \text{and} \quad M_0 = 4tR^2\sigma_y$$

To calculate the Tresca case recall that for a thin cylinder we can ignore the radial stress compared with the axial and hoop stresses, so we approximate $\sigma_{radial} = 0$. A lower bound solution is developed by putting the hoop stress to $\sigma_h = \frac{PR}{t}$ and the axial stress to $\sigma_a = \frac{1}{2}\sigma_h \pm \sigma_t$, where the positive/negative signs apply on the upper/lower halves of the section. Hence, on the upper half the Tresca stress is $\sigma_h - \sigma_{radial} = \sigma_h$ whereas on the lower half the Tresca stress is either this, σ_h, or $\sigma_h - \sigma_a = \frac{1}{2}\sigma_h + \sigma_t$, whichever is the larger. Hence the Tresca criterion is,

$$\sigma_{Tresca} = MAX\left(\frac{1}{2}\sigma_h + \sigma_t, \sigma_h\right) \le \sigma_y$$

But to balance the applied moment we require $M = 4tR^2\sigma_t$, so this criterion becomes,

$$\frac{PR}{t} \le \sigma_y \quad \textbf{and} \quad \frac{1}{2}\cdot\frac{PR}{t} + \frac{M}{4tR^2} \le \sigma_y$$

Giving,
$$\frac{P}{P_0} \le 1 \quad \textbf{and} \quad \frac{1}{2}\cdot\frac{P}{P_0} + \frac{M}{M_0} \le 1$$

So the interaction is bi-linear, thus,