Tutorial Session 24: T73S06 (Creep Rupture / R5V6) -
Formulation of the Structural Analysis Problem in Creep

Relates to Knowledge & Skills items 1.3, 2.2 and aspects of T73S03 & T73S04.

Much of this session is background material which is implicitly assumed in the creep SQEP roles. However, there should have been an explicit question on creep hardening laws in the Mentor Guides, initially my omission which should be remedied.

Last Update: 15/7/14

Formulation of the structural analysis problem in creep (in brief); Examples of structural response in creep; Distinct creep responses to primary and secondary loads; Relaxation versus redistribution; Time variation of stress distribution across a notched bar; Rupture reference stress; Concept of skeletal point/stress; Neuber relation; Creep hardening laws: relevance to changes in conditions; Reminder of the scope of the various Volumes of R5

Qu.: How is the elastic structural analysis problem formulated?

In T72S01 Session 7 we saw how the general elasticity problem could be formulated. Whilst it is rare that exact, analytical solutions can be derived, nevertheless it is important to understand the equations which govern the structural response. These are the equations which the computer is implicitly solving when you do a finite element analysis. In 3D elasticity these equations are,

Equilibrium: $\sigma_{ij,j} = -b_i$ ($\bar{b}$ = applied load per unit volume) (1)

Hooke's Law (elasticity): $\varepsilon_{ij} = C_{ijkl}\sigma_{kl}$ (2)

Compatibility: $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ OR $F(\varepsilon'') = 0$ (3)

The compatibility condition ensures that there is an underlying displacement field, $\bar{u}$, which is compatible with the strain field. This is imposed either by the explicit expression for the strains in terms of the displacements, or by use of the compatibility equations, $F(\varepsilon'') = 0$ which are mathematically equivalent to $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$. The compatibility equations are second order differential equations in the strains. The form of the equations depends upon the coordinate system. For a 2D problem in Cartesian coordinates they reduce to just one equation, namely,

2D: $\varepsilon_{xx,yy} + \varepsilon_{yy,xx} - 2\varepsilon_{xy,xy} = 0$ (3b)

Qu.: How can stresses and strains be found in creep?

The equilibrium equation is the same whether creep is present or not. Hooke's Law no longer holds in creep. However, the *elastic part* of the strain, ε_{ij}^e, is defined as having the usual linear relationship with stress. Finally, the *total* strain, ε_{ij}, is still required to be geometrically compatible with a displacement field, so the compatibility equations are unchanged so long as it is understood to involve the *total* strain. Hence the above three equations remain true except for the replacement of strain for elastic strain in the second equation,

Equilibrium: $\sigma_{ij,j} = -b_i$ ($\bar{b}$ = applied load per unit volume) (4)

Definition of elastic strains: $\varepsilon_{ij}^e = C_{ijkl}\sigma_{kl}$ (5)

Compatibility: $$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \text{ OR } F(\varepsilon'') = 0 \quad (6)$$

However, this is no longer a soluble set of equations because we have as yet no way of relating the *total* strain, ε_{ij}, to the stress. Of course, by the total strain we mean the sum of the elastic and creep strains, so we can write,

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{c} \quad (7)$$

But Equs.(4-7) are not a soluble set because there is no way to find the creep strains.

Qu.: What is the missing ingredient?

The ingredient which is missing is a means of calculating the creep strain. This is provided, of course, by a creep deformation equation. These equations are developed for uniaxial creep, but the uniaxial stress and creep strain are then assumed also to hold for the equivalent (e.g., Mises) quantities in multi-axial creep. (At least if the material is isotropic).

Qu.: What does a creep deformation law look like?

Creep deformation laws specify the instantaneous equivalent creep strain ***rate*** as a function of equivalent stress, temperature and some 'state variable', for example,

Time hardening: $$\dot{\bar{\varepsilon}}^{c} = f(\bar{\sigma}, T, t) \quad (8a)$$

Strain hardening: $$\dot{\bar{\varepsilon}}^{c} = f(\bar{\sigma}, T, \bar{\varepsilon}^{c}) \quad (8b)$$

General hardening law: $$\dot{\bar{\varepsilon}}^{c} = f(\bar{\sigma}, T, \lambda) \quad (8c)$$

To date, strain hardening has been the most common assumption in assessments. However, it is not well founded in terms of experimental justification – especially for austenitic materials, and especially when operating temperatures have changed. Work is underway to recommend a more soundly based hardening law. This essentially comes down to defining just what the state variable λ actually is. In general it would be specified as a rate,

$$\dot{\lambda} = g(\bar{\sigma}, T, t, \bar{\varepsilon}^{c}, D_{c}, ...) \quad (9)$$

where D_c is creep damage (whatever that is!). This general formulation subsumes primary, secondary and tertiary behaviour.

Here we have pretended that there is just one scalar state variable, λ. But there may be many state variables, really, and they may not be scalars. These state variables are just a means of accounting for the effects of all those complicated metallurgical mechanisms which we talked about in session 23.

Qu.: What are the individual components of creep strain rate?

The individual components of creep strain rate can be found in terms of the equivalent strain rate by using the normality rule with the "yield" (i.e., creep) surface of your choice. For a Mises material we have,

$$\dot{\varepsilon}_{ij}^{c} = \frac{3}{2} \cdot \frac{\hat{\sigma}_{ij}}{\bar{\sigma}} \dot{\bar{\varepsilon}}^{c} \quad (10)$$

Qu.: So what is the complete set of creep equations?

The complete set of equations defining the structural analysis problem in creep is equations (4), (5), (6), (7), one of (8), plus (9) if required, and finally (10).

Simple!

Well, ok, not simple. And not amenable to analytical solution in transient creep conditions in non-trivial cases.

Qu.: How does creep affect simple uniaxial tension?

If the uniaxial tensile stress is kept constant, then the only thing varying is the strain – and this follows simply by integrating the relevant one of Equs.(8).

If the uniaxial *load* is constant, and the strains remain small, then this approximates to constant stress and the same applies.

However, if the loading is strain controlled, i.e., if a fixed displacement is applied, then the load will relax.

Qu.: Relaxation in simple uniaxial displacement control assuming time hardening

Consider a bar or plate with a displacement applied and held fixed. The initial elastic strain, ε_0, is just the applied displacement divided by the length of the bar/plate. As creep strain accumulates, the total strain remains fixed,

$$\varepsilon^e + \varepsilon^c = \varepsilon_0 \tag{11}$$

So, if the creep strain, ε^c, is increasing, then the elastic strain, ε^e, must be decreasing. But in uniaxial stress, the elastic strain is related to the stress simply by $E\varepsilon^e = \sigma$. So the stress must be relaxing due to creep. We can thus re-write (11) as,

$$\frac{\sigma}{E} + \varepsilon^c = \varepsilon_0 \qquad \text{i.e,} \qquad \sigma = E\left(\varepsilon_0 - \varepsilon^c\right) \tag{12}$$

Suppose we assume a creep deformation law of the form,

$$\dot{\varepsilon}^c = C \cdot t^m \cdot \sigma^n \tag{13}$$

This is a primary creep law, since the strain rate has a time-dependence. But a secondary creep law results when $m = 0$. Substituting (12) into (13) gives,

$$\dot{\varepsilon}^c = CE^n \cdot t^m \cdot \left(\varepsilon_0 - \varepsilon^c\right)^n \tag{14}$$

If we assume time hardening, Equ.(14) can be integrated, i.e.,

$$\int_0^{\varepsilon^c} \frac{d\varepsilon^c}{\left(\varepsilon_0 - \varepsilon^c\right)^n} = CE^n \int_0^t t^m\, dt \tag{15}$$

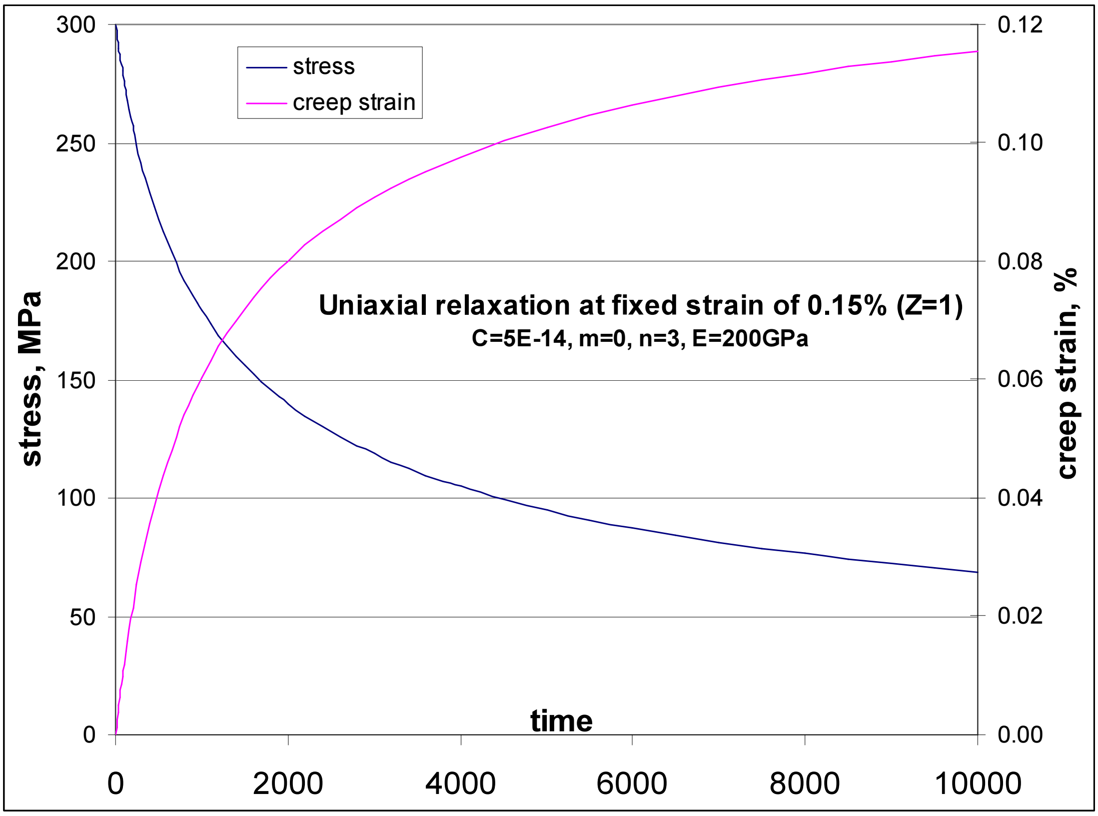
With the result that the relaxing stress is given by,

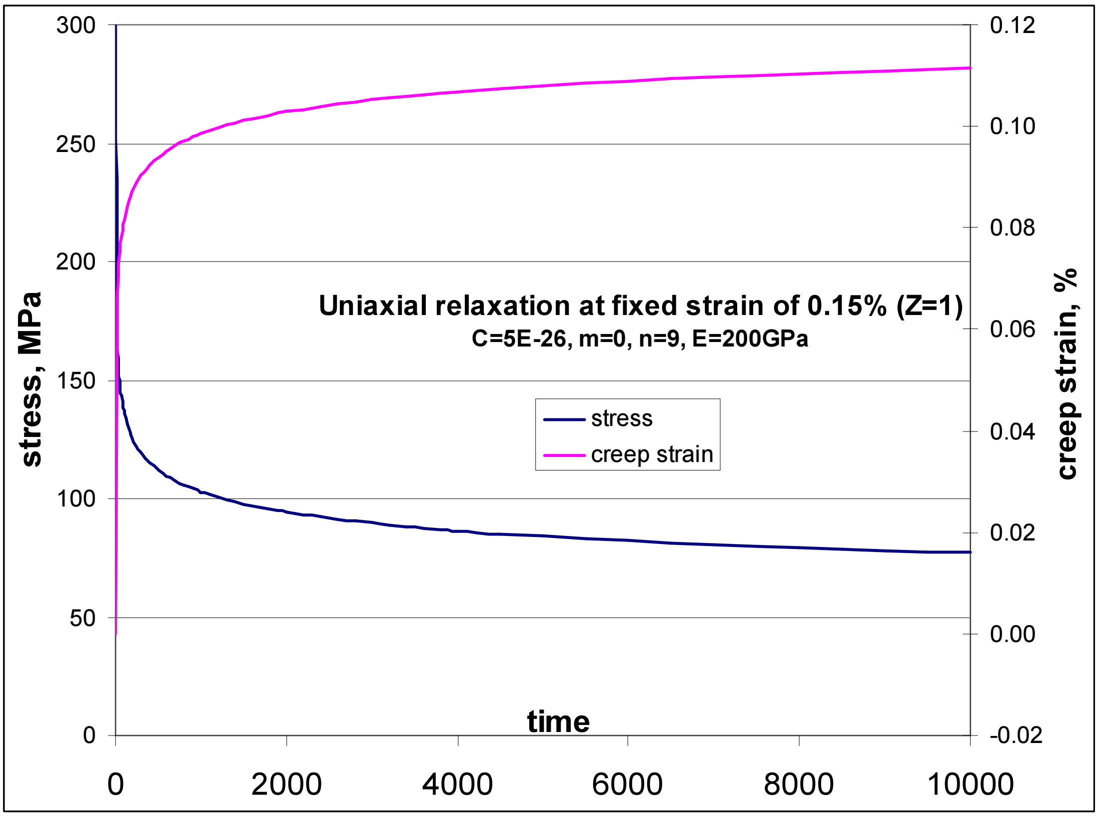
$$(n > 1) \qquad \sigma = E\left\{\varepsilon_0^{-(n-1)} + \left(\frac{n-1}{m+1}\right) CE^n t^{m+1}\right\}^{-\frac{1}{n-1}} = \left\{\sigma_0^{-(n-1)} + \left(\frac{n-1}{m+1}\right) CEt^{m+1}\right\}^{-\frac{1}{n-1}} \tag{16}$$

The creep strain then follows from (12). From a starting stress of 300 MPa (i.e., a fixed strain of 0.15% for a material with E = 200 GPa), Figures 1 and 2 illustrate the relaxation curve which results for two material behaviours. Both are for pure secondary creep, i.e., $m = 0$. Figure 1 is for a material with $n = 3$, whereas Figure 2 has a much more stress-sensitive creep rate with $n = 9$.

By 10,000 hours both materials have relaxed to a similar extent, acquiring almost the same creep strain (because I have fiddled the values of C so that this is the case). However, the $n = 9$ material relaxes far faster initially, with most of the relaxation occurring in the first 100 hours. In contrast, the $n = 3$ material relaxes only slightly in 100 hours. Consequently these two materials might behave similarly for 10,000 hour dwells, but very differently for 100 hour dwells.

Both these n values are quite realistic and do occur for real materials.





Qu.: Can a time-hardening creep law be re-expressed as strain-hardening?

Yes, but only if the conditions of stress and temperature are constant. Usually a creep deformation law in time-hardening form, such as,

$$\dot{\varepsilon}^c = C \cdot t^m \cdot \sigma^n \tag{17}$$

is derived from a constant stress test (or a constant load test approximating constant stress for small strains). So, for a given stress, (17) is derived from the slope of the strain versus time curve, and how this varies with time. Note that the usual primary creep behaviour has a reducing slope, so that $m < 0$.

WARNING: Some deformation laws, such as RCC-MR for 316ss, are formulated to give the creep strain, rather than the strain rate, i.e., $\varepsilon^c = C_1 \cdot t^{C_2} \cdot \sigma^{n_1}$. In this case primary creep is given by $0 < C_2 < 1$. Differentiating wrt time shows that $m = C_2 - 1$ and $C = C_1 C_2$.

Provided that the stress is held constant, (17) can be integrated to give the creep strain, i.e., $\varepsilon^c = \frac{C}{m+1} \cdot t^{m+1} \cdot \sigma^n$. Using this expression we can find time in terms of the creep strain. When substituted into (17), the strain rate can be written in the alternative form,

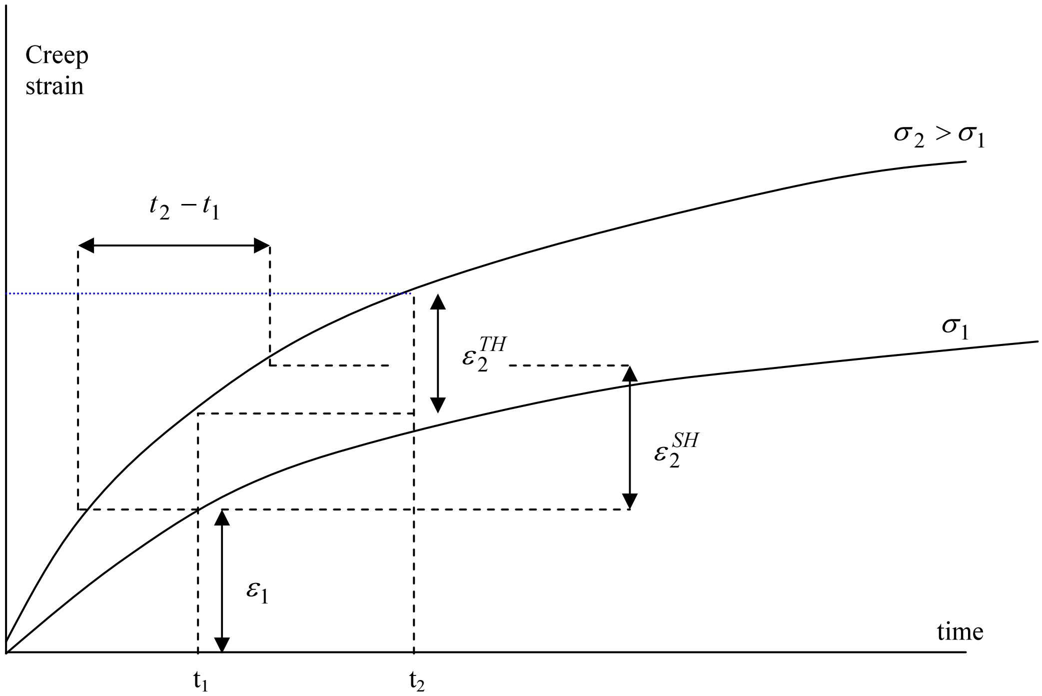
$$\dot{\varepsilon}^c = \tilde{C} \cdot \left(\varepsilon^c\right)^{\tilde{m}} \cdot \sigma^{\tilde{n}} \tag{18}$$

where, $$\tilde{C} = \left[(m+1)^m C\right]^{\frac{1}{m+1}}, \quad \tilde{m} = \frac{m}{m+1} \text{ and } \tilde{n} = \frac{n}{m+1} \tag{19}$$

However, it is crucial to appreciate that…

A creep deformation law in time hardening form, e.g., (17), is equivalent to one expressed in strain hardening form, e.g., (18), ***only if*** the conditions of stress and temperature remain constant. In general (17) and (18) will give different predictions for the stress and creep strain if either stress or temperature vary.

Qu.: How do time hardening and strain hardening differ when conditions vary?



The above sketch graph depicts the creep strain versus time curve for two different stress levels, assuming constant stress. Consider maintaining a stress of σ_1 for a time t_1, and then increasing the stress to σ_2 until time t_2. What is the total creep strain at time t_2? Clearly it must be less than that indicated by the blue dashed line in the Figure since that is what would occur if the higher stress were sustained over the whole time period.

Over the first time period (and assuming the material was in the virgin state before that), the two creep laws, (17) and (18), both imply the same creep strain, ε_1. This is given by,

$$\varepsilon_1 = \frac{C}{m+1} \cdot t_1^{m+1} \cdot \sigma_1^{n} = \left[(1-\tilde{m})\tilde{C} t_1 \sigma_1^{\tilde{n}}\right]^{\frac{1}{1-\tilde{m}}} \tag{20}$$

For time hardening the strain rate just after time t_1 is found by moving vertically upwards (i.e., at the same *time*) to the creep curve for stress σ_2. The creep strain increment in the second period is thus given by ε_2^{TH}, as shown in the Figure . By integrating (17) from t_1 to t_2 we find,

$$\varepsilon_2^{TH} = \frac{C}{m+1}\left(t_2^{m+1} - t_1^{m+1}\right)\sigma_2^{n} \tag{21}$$

And hence the total creep strain by time t_2 using time hardening is,

$$\varepsilon_1 + \varepsilon_2^{TH} = \frac{C}{m+1}\left\{t_1^{m+1} \cdot \sigma_1^{\ n} + \left(t_2^{m+1} - t_1^{m+1}\right)\sigma_2^n\right\} \tag{21b}$$

In contrast, the creep accumulated in the second period according to the strain hardening assumption is derived by moving horizontally to the left (i.e., at the same *strain)* to the creep curve for stress σ_2. The creep strain increment in the second period is thus given by ε_2^{SH}, as shown in the Figure . By integrating (18) from t_1 to t_2 we find the total strain due to the first and second periods to be,

$$\varepsilon_1 + \varepsilon_2^{SH} = \left[(1-\tilde{m})\tilde{C}t_1\sigma_1^{\tilde{n}} + (1-\tilde{m})\tilde{C}(t_2 - t_1)\sigma_2^{\tilde{n}}\right]^{\frac{1}{1-\tilde{m}}} = \frac{C}{m+1}\left[t_1\sigma_1^{\tilde{n}} + (t_2 - t_1)\sigma_2^{\tilde{n}}\right]^{m+1} \tag{22}$$

The two expressions, (21b) and (22), are clearly not equal in general.

Qu.: Which is more conservative, strain hardening or time hardening?

It is evident from the Figure that strain hardening will produce the larger creep strain in this example, i.e., when the stress is increased. Conversely, if the stress in the second period were smaller than in the first period, strain hardening would lead to the smaller creep strain.

An illustration is as follows. Suppose $C = 3.125 \times 10^{-14}$, $m = -0.5$, $n = 4$. Consider 5000 hours at 150 MPa followed by a further 5000 hours at 200 MPa. Substitution in the above expressions produces a total creep strain assuming time hardening of 0.52%, whereas assuming strain hardening gives 0.74%.

The difference is substantial and emphasises the importance of the assumed hardening law.

Applying the two stresses in reverse order, we would get a total creep strain assuming time hardening of 0.80%, whereas assuming strain hardening again gives 0.74%. The difference is smaller in this case because the second period contributes the smaller amount to the strain.

Qu.: In the last example, for strain hardening the final creep strain was independent of the order in which the stresses were applied. Is this always the case?

No.

This comes about because the duration of the first and second periods was assumed to be the same (both 5000 hours). Strain hardening always gives the same final strain, independent of the order of the stresses, providing that the time intervals are equal. This can be seen in the general case as follows. Consider the loading sequence,

Stress σ_1 applied between time 0 and time t_1;

Stress σ_2 applied between time t_1 and time t_2;

Stress σ_3 applied between time t_2 and time t_3, etc.;

The final strain is,

<u>Time Hardening:</u>

$$\varepsilon_f^{TH} = \frac{C}{m+1}\left\{t_1^{m+1}\sigma_1^n + \left(t_2^{m+1} - t_1^{m+1}\right)\sigma_2^n + \left(t_3^{m+1} - t_2^{m+1}\right)\sigma_3^n + ...\right\} \tag{23}$$

Strain Hardening:

$$\varepsilon_f^{SH} = \frac{C}{m+1}\left\{t_1\sigma_1^{\tilde{n}} + (t_2 - t_1)\,\sigma_2^{\tilde{n}} + (t_3 - t_2)\,\sigma_3^{\tilde{n}} + \ldots\right\}^{m+1} \quad (24)$$

But if all the time intervals are equal, i.e., if $t_{i+1} - t_i = \tau$, then the strain hardening result becomes,

$$\varepsilon_f^{SH} = \frac{C}{m+1}\tau^{m+1}\left\{\sigma_1^{\tilde{n}} + \sigma_2^{\tilde{n}} + \sigma_3^{\tilde{n}} + \ldots\right\}^{m+1} \quad (25)$$

It is clear from (25) that the same final strain would result whatever the order in which the stresses $\sigma_1, \sigma_2, \sigma_3, \ldots$ were applied. Hence,

For equal time intervals and constant temperature, the creep strain resulting from strain hardening does not depend upon the order in which the stresses are applied.

Although we have proved this only for the case of a power-law deformation law, $\dot{\varepsilon}^c = \tilde{C}\cdot(\varepsilon^c)^{\tilde{m}}\cdot\sigma^{\tilde{n}}$, it is simply shown for any deformation law in which strain and stress are separable: $\dot{\varepsilon}^c = f(\varepsilon^c)g(\sigma)$. In this case, N equal intervals of time τ produce a final creep strain, assuming strain hardening, of $\varepsilon_f^{SH} = F^{-1}\left(\tau\sum_{i=1}^{N} g(\sigma_i)\right)$, where F is the integral of $1/f$, and F^{-1} is its inverse function. The form of this result shows that it is independent of the order of the stresses $\{\sigma_i\}$.

If the time intervals were different, the order would matter.

For time hardening the order would matter even with equal time intervals. In time hardening, the order would not influence the result if the times were such that $t_{i+1}^{m+1} - t_i^{m+1} = t_1^{m+1}$. For example, for $m = -0.5$, time hardening would be order independent for times chosen to be 1, 4, 9, 16… times the first time interval.

Qu.: When are strain hardening and time hardening the same?

Strain hardening and time hardening make no difference if conditions of stress and temperature are unchanging.

But also note that the difference between strain hardening and time hardening comes about because the creep versus time graph is curved. If it were a straight line, then there would be no difference. But even if a structure is currently in secondary creep, a historical period of primary creep will permanently affect the creep strain accumulated.

[1]	Time hardening and strain hardening are the same if stress and temperature are constant.
[2]	Time hardening and strain hardening are the same if secondary creep applies from t=0 even if conditions vary.

Qu.: What is the difference between relaxation and redistribution?

Relaxation applies only to secondary stresses. It refers to the reduction of a net load (force or moment) due to creep strain accumulation. This can only happen if the loading is displacement controlled or strain controlled.

Redistribution, on the other hand, can apply to any type of loading. It relates to a situation in which the stress is non-uniform across a section. Because creep rates tend to vary as some large power of stress, the creep strain will accumulate far faster in the region of higher stress. The effect of creep is therefore to preferentially reduce the magnitude of the larger stresses. This can occur even for primary loads, in which the net force and moment are constant, so long as there are compensating stress increases at other parts of the section. Hence, the effect of creep is to *redistribute* the stress from the higher to the lower stressed regions.

Redistribution will tend to make the stress across the section more uniform than its initial distribution. But the stress distribution does not necessarily become flat (membrane) even after very long times. This may be the case, but not in general. In general, and especially if there is a stress concentration feature, the stress will remain elevated in that region even when the creep achieves steady conditions and the stresses are no longer changing. This is called the "steady creep stress distribution".

Secondary stresses will be subject to both redistribution and relaxation. The timescale for redistribution will tend to be shorter than for relaxation, because, by definition, redistribution relates to creeping of the most highly stressed part of the section. In contrast, relaxation of the net load is driven by the effective 'reference' stress.

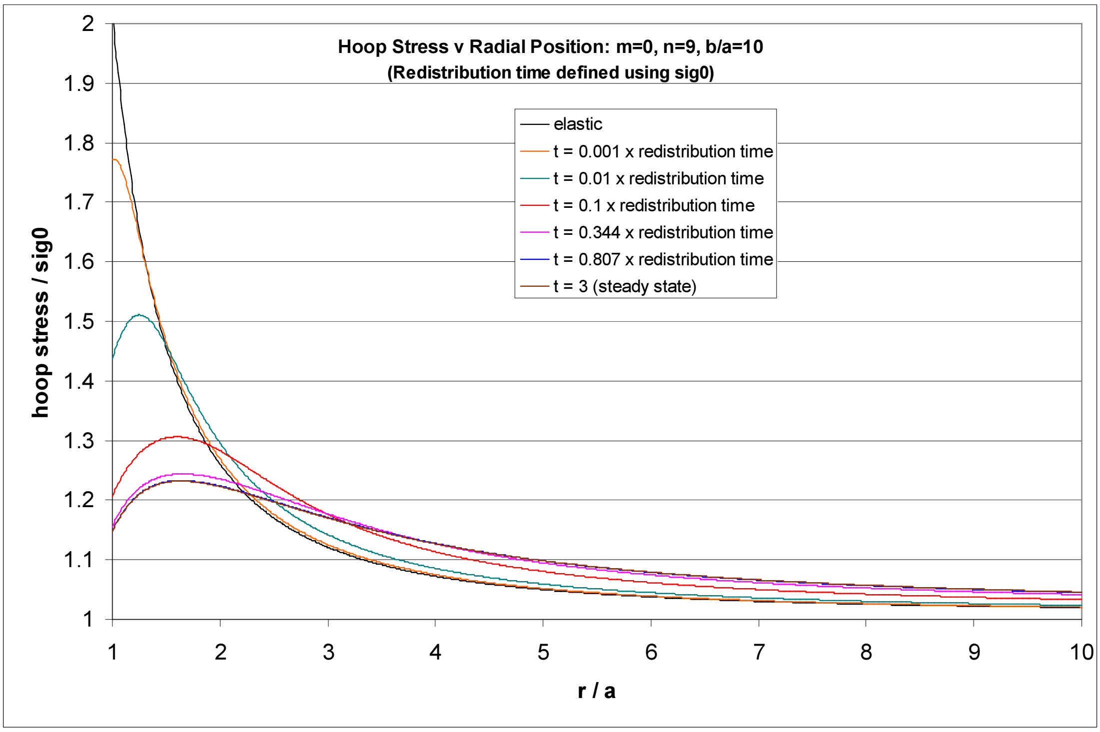
Qu.: Is there a simple example of redistribution behaviour?

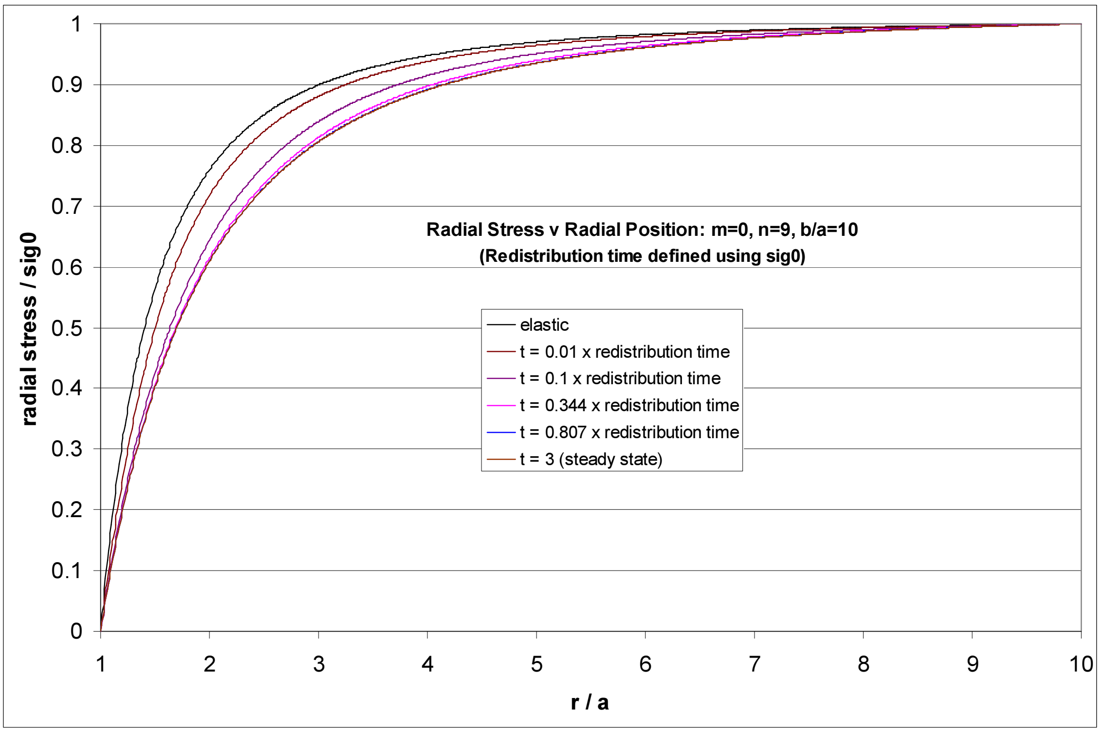
The example we shall employ is a flat annular plate of outer radius b containing a concentric hole of radius a assuming plane stress conditions. A radial tensile stress, σ_0, is applied around the whole of the outer circumference. For $b \to \infty$ this would be equi-biaxial tension remote from the central hole. For finite b it will differ slightly from equi-biaxial tension since the hoop stress at the OD will not be exactly equal to the applied radial stress. The solution has been derived numerical. The complete equation set and the solution algorithm are given in Appendix A.

The method can be used with any User supplied creep deformation law. Here we shall give illustrative results for secondary creep and a Norton law, i.e., as Equs.(17) or (18) but with $m = 0$.

Example results below are for $b/a = 10$, $n = 9$, $C = 5 \times 10^{-26}$ and $\sigma_0 = 100$ MPa. Time has been normalised by 10,000 hours.

Example results for annular plate with applied remote radial stress





Because b/a is large, the results approximate to applied equi-biaxial tension. Hence, the elastic hoop stress at the hole is ~2 times the applied stress (i.e., the SCF for a circular hole in an equi-biaxial stress field is 2) – see top graph. However, this peak stress relaxes rapidly in only tens or hundreds of hours. By ~4,000 hours the stresses achieve a steady distribution which does not change thereafter. This is steady creep.

Note that the hoop stress near the hole is still elevated above the applied stress. The hoop stress is now greatest sub-surface, with a concentration factor of 1.23.

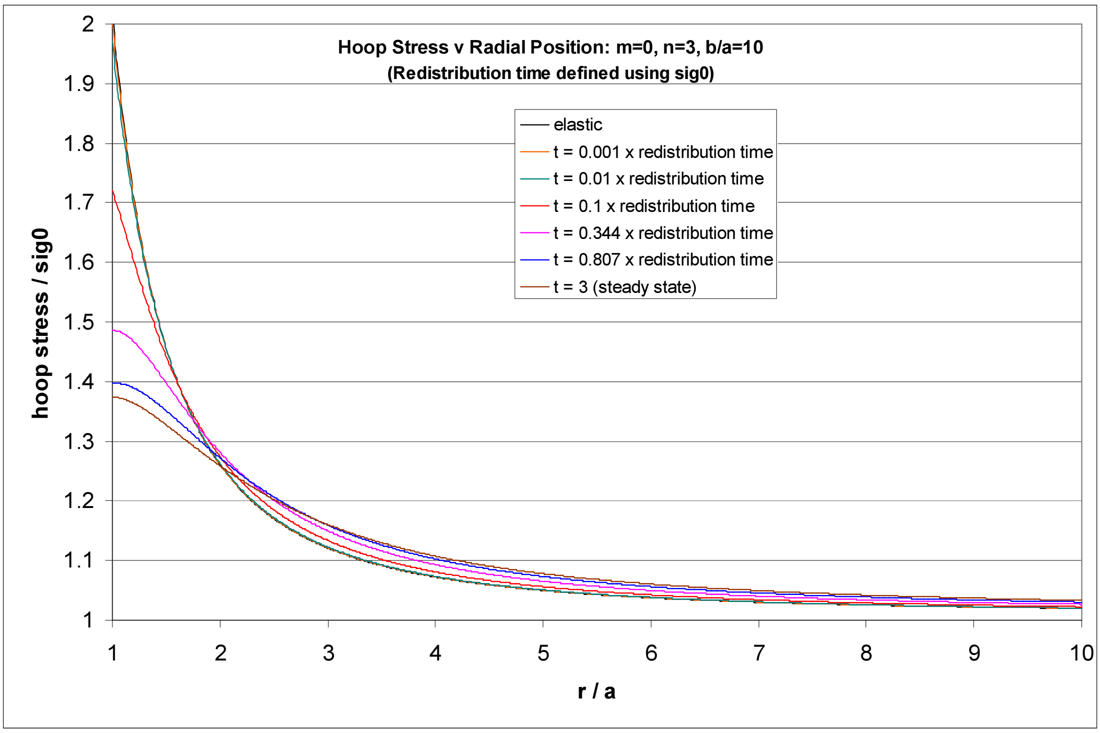
Note that whilst the stresses near the hole have decreased substantially, the stresses over the bulk of the section away from the hole have increased modestly. The average hoop stress is unchanged throughout, due to the requirement to equilibrate the applied load. (The average hoop stress is always 100/(1-a/b) = 111 MPa, i.e., 1.11 when normalised).

This illustrates the *redistribution* of stresses whilst retaining a constant load.

It also illustrates that the stresses do not redistribute so much that the distribution becomes flat, however long you wait.

Qu.: How sensitive is the behaviour to the creep index, *n*?

The variation of the hoop stress distribution when n is reduced to 3 (with C = $5x10^{-14}$) is shown below.



The behaviour is qualitatively similar, but the peak stress now remains on the hole surface. The steady state hoop stress distribution has a larger peak value (1.37) compared with $n = 9$.

Qu.: What do the steady state stress distributions imply regarding creep rupture?

The key point to note is that, because the hoop stress distribution does not become completely flat, it would be incorrect to use the ordinary "plastic" reference stress to assess creep rupture.

What is the ordinary reference stress for this problem? It is simple to show that the Tresca collapse solution has simply a uniform hoop stress equal to the yield stress (proved in Appendix B), so the average hoop stress is also the reference stress. Hence, $\langle \sigma_H \rangle = \sigma_{ref} = \dfrac{\sigma_0}{1 - a/b} = 1.11\sigma_0$ for $b/a = 10$.

But we have seen that the peak hoop stress (which is also the peak Tresca stress in plane stress) is $1.23\,\sigma_0$ (for $n = 9$) or $1.37\,\sigma_0$ (for $n = 3$). Hence, there is a peak Tresca equivalent stress in excess of the Tresca reference stress which persists indefinitely in the steady state. This point will accumulate creep strain (damage) faster than it would at the reference stress. It would therefore be potentially non-conservative to base creep rupture on the ordinary plastic limit-state reference stress.

> In general, the stress distribution in steady state creep will have points at which the equivalent stress exceeds the ordinary "plastic" reference stress.

> Consequently it would potentially be non-conservative to base an assessment of creep rupture on the ordinary "plastic" reference stress.

Qu.: What is the "rupture reference stress"

The rupture reference stress is an enhanced reference stress which takes account of any persistent stress elevation, even in steady state creep, and can be used to assess creep rupture.

Creep rupture is assessed against primary loads only (R5 V2/3 Section 6.5). The first step is to calculate the ordinary reference stress, σ_{ref}. This is the same primary load reference stress which is used in R6 and the same sources can be used to find suitable solutions.

R5 V2/3 Section 6.5 and Appendix A5 define a rupture reference stress as,

Creep ductile materials: $$\sigma_{ref}^R = \{1 + 0.13[\chi - 1]\}\sigma_{ref} \quad (26a)$$

Creep brittle materials: $$\sigma_{ref}^R = \left\{1 + \frac{1}{n}[\chi - 1]\right\}\sigma_{ref} \quad (26b)$$

where n is the creep index (the power of stress in the deformation law). The term χ is a form of stress concentration factor, defined as the ratio of the maximum primary load elastic equivalent stress to the reference stress: $\chi = \bar{\sigma}_{el,\max} / \sigma_{ref}$.

We can apply this R5 prescription to our annular plate example to see how well it performs. The material in question might be creep ductile or creep brittle, so both options are used. The maximum elastic equivalent stress in this example is $\sim 2\sigma_0$, and the reference stress is $\sigma_{ref} = 1.11\sigma_0$, so that $\chi = \bar{\sigma}_{el,\max} / \sigma_{ref} = 2/1.11 = 1.80$. This gives the following R5 rupture reference stresses, normalised by σ_0,

$\sigma_{ref}^{R} / \sigma_0$	**n = 3**	**n = 9**
Creep Ductile	1.23	1.23
Creep Brittle	1.41	1.21
Annular Plate: *Tresca* *Mises*	 *1.37* *1.37*	 *1.23* *1.15*

The annular plate results quoted above are simply the peak equivalent stress in steady state creep. The R5 rupture reference stresses compare reasonably well with the analytic result for this geometry and loading (particularly if n = 3 is associated with creep brittle behaviour).

Qu.: How is creep rupture assessed?

Having found the rupture reference stress, creep rupture is assessed by inserting this stress into the appropriate time-to-rupture expression.

Note, however, that if there is substantial compression, or substantial triaxiality, then special rupture expressions may apply which depend upon all three stress invariants, not just the equivalent stress, see R5 V2/3 Appendix A5. Relevant references are,

[1] Hayhurst, D. R. (1972) Creep rupture under multiaxial states of stress. J. Mech. Phys. Solids 20, 381–390.

[2] Hayhurst D.R., Dimmer P.R. and Morrison C.J. (1984). Development of continuum damage in the creep rupture of notched bars. Phil. Trans. R. Soc. London, A311, 103-129

[3] R.L.Huddleston, "Assessment of an improved multiaxial strength theory based on creep-rupture data for type 316 stainless steel", ASME J.Pres.Ves.Tech. 115, 177-184 (1993). See also R5V2/3 Appendix A5, Section A.5.2.2.1; See also Huddleston, R. L. (1985) An improved multiaxial creep rupture strength criterion. ASME J. Press. Vess. Technol. 107, 421–429.

[4] Perrin I.J. and Hayhurst D.R. (1999). Continuum damage mechanics analysis of type IV creep failure in ferritic steel crossweld specimens. Int. J. Press. Vess. and Piping. 76, 599-617

[5] A.Baker, "Multiaxial creep rupture of Type 316H steel", E/REP/BDBB/0066/GEN/05, October 2005.

Qu.: What is a "skeletal point"?

When considering the transient creep due to a primary load near a stress concentrator, we have seen that the peak stresses reduce whilst the stresses away from the SCF increase. Comparing any two times there will therefore be a point whose stress has remained unchanged (where the two curves cross). It is sometimes claimed that this "invariant point" remains at approximately the same position for all times. If so, it is known as a "skeletal point". This concept is of utility when analysing the results of creep tests on notched bars – a common means of obtaining creep data under triaxial conditions.

But is there really an invariant skeletal point? For our biaxial annular plate the approximation is not terribly good. For $n = 3$, there is a rough approximation to a skeletal point in the hoop stress at $r/a \sim 2$. For $n = 9$, the crossing points of the curves of hoop stress tend to move outwards to larger *r*. After about 10% of the redistribution time, there is a reasonably fixed skeletal point at *r/a* ~ 3.1.

I believe that other geometries and loadings, e.g., a notched bar in tension, produce rather better skeletal points. An example below is from a paper by Hayhurst & Dyson,

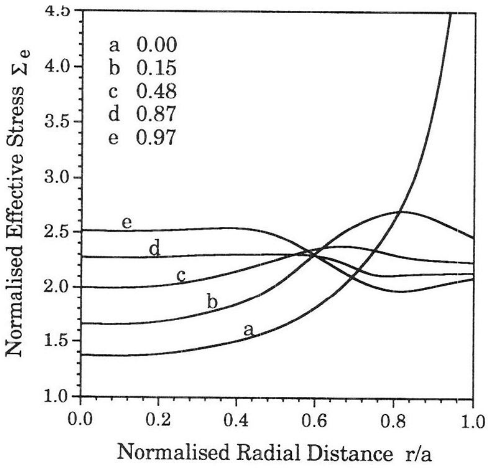


Fig. 2. Spatial variation of normalised effective stress (Σ_e) across the notch throat for different values (indicated on the graph) of the normalised time t/t_n determined for $\sigma_N = 425$ MPa with $C = 300$, $D = 2$ and $v = 4$.

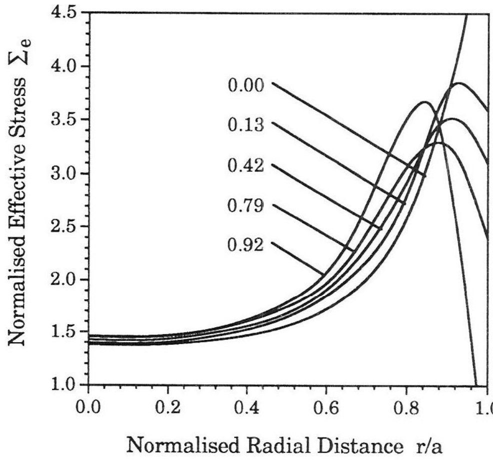
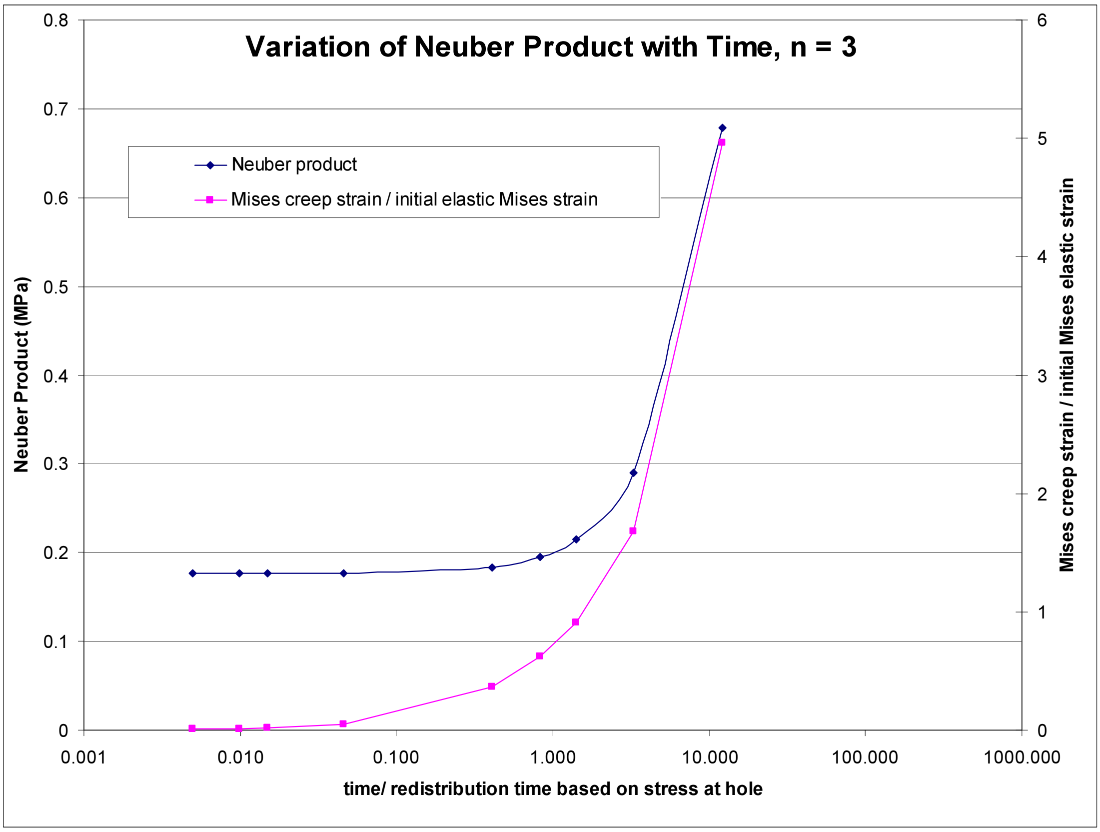


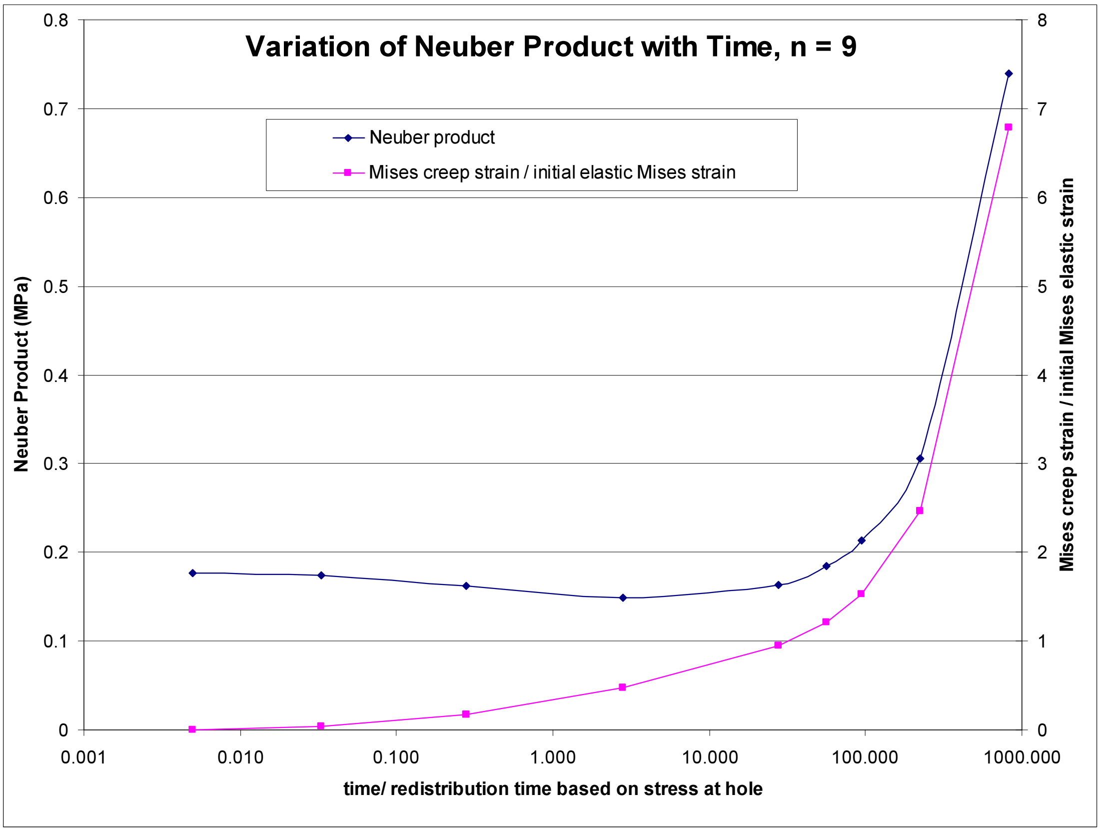
Fig. 3. Spatial variation of normalised effective stress (Σ_e) across the notch throat for different values (indicated on the graph) of the normalised time t/t_n determined for $\sigma_N = 425$ MPa with $C = 300$, $D = 32$ and $v = 4$.

The position of the skeletal point, and degree to which it is well defined, varies with the material creep response. The curve on the left has a creep ductility of 15% whereas that on the right has a little less than 1%. In the latter case redistribution is far from complete when the specimen initiates a crack at the notch.

Qu.: Does creep redistribution respect the Neuber relation?

Neuber's rule holds that the product of the equivalent stress and the equivalent strain near a notch is the same when calculated elastically as when calculated for some non-linear material response. Usually the "non-linear material response" is interpreted as meaning "plasticity". In R5 V2/3, the Neuber relation is the basis for constructing the elastic-plastic stress-strain hysteresis cycles from a knowledge of the elastic stresses alone. R5 does *not* employ the Neuber rule in the context of creep (instead an elastic follow-up factor, Z, is used). However, it is of interest to see if the Neuber rule does work for the redistribution of stresses during transient creep. The graphs below plot the results of our annular plate example, for $n = 3$ and $n = 9$ respectively.





The blue lines are the product of the equivalent stress and strain, plotted against a normalised time. The pink curves are the equivalent creep strain accumulated, normalised by the elastic strain σ_0 / E . We see that the Neuber product, $\overline{\sigma\varepsilon}^T$, is indeed reasonably constant so long as the creep strain remains less than the elastic strain σ_0 / E . Thereafter, the Neuber product climbs steeply. This is inevitable, of course, since once steady creep is approached the stress will remain constant whilst the creep strain continues to accumulate at a constant rate. Nevertheless it is of interest to see that the Neuber relation holds reasonably well during the transient redistribution phase.

Qu.: What are the various Volumes of R5 for?

Volume 1: Overview.

Volume 2/3: *Creep-fatigue crack initiation for initially defect free structures*. It used to be split into two volumes, with V2 providing guidance on the analysis (e.g., cycle construction) and V3 dealing with the assessment of damage. Now it is rationalised into just one volume. This volume also includes guidance on various 'precursor' assessments, necessary for the main "initiation" assessment to be valid. This includes various code-like stress limits, shakedown/ratcheting assessment, creep rupture and cyclic creep assessments. It includes specific guidance on crack initiation assessment of weldments (Appendix A4). The latter is to undergo a major revision when R5V2/3 Issue 3 Revision 2 is issued later this year (2014). Volume 2/3 is the subject matter of SQEP sub-role T73S04.

Volume 4/5: *Creep-fatigue crack growth*. Perhaps the most significant part of this volume is the methodology for calculating the parameter C(t) used to calculate creep crack growth rates. This volume uses the concept of "total reference stress", or pseudo-reference stress, which includes secondary loads as well as primary loads. V4/5 used to be two volumes, with V4 being for structures in which cyclic loading did not significantly influence creep crack growth (and V5 being for those where it did). These two procedures are now rationalised into the one volume. In May 2012 R5V4/5 Issue 3 was re-issued at Revision 1. The most significant changes introduced by Revision 1 were in the C(t) estimation under mixed primar+secondary loading, specifically to, (i)the evaluation of total reference stress when secondary stresses are relaxing and the crack is growing sufficiently to significantly influence the reference stress; and, (ii) the evaluation of total reference stress including secondary stresses with large out-of-plane components. This volume also provides the methodology for calculating creep-fatigue crack growth rates in dissimilar metal welds (should you be fortunate enough to have the requisite materials data, which is unlikely at present). Volume 4/5 is the main subject matter of SQEP sub-role T73S03.

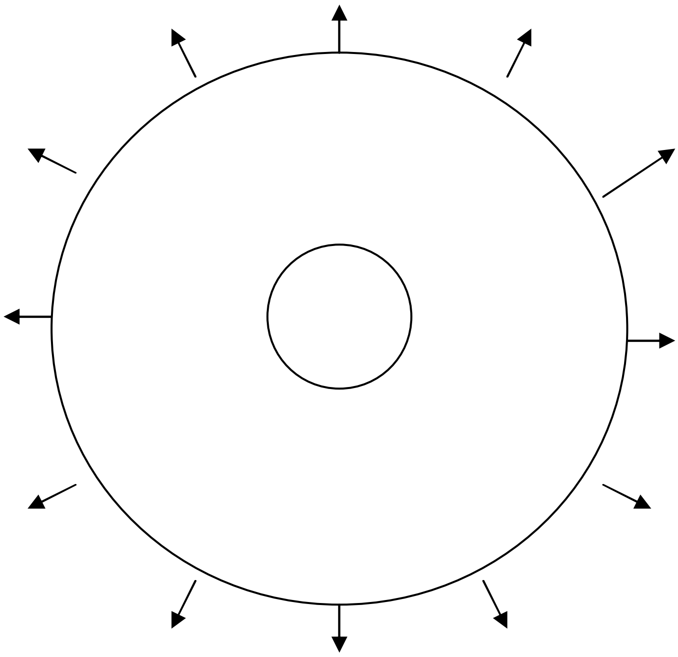
Volume 6: *Assessment of defect-free dissimilar metal welds*. Dissimilar metal welds should be assessed using V6 rather than the general purpose procedure, V2/3. It is conservative to regard V6 as a "failure" assessment, as opposed to a crack initiation assessment, though there are elements of "initiation" about it. Volume 6 is included within the subject matter of SQEP sub-role T73S06.

Volume 7: *Ferritic similar metal welds under steady creep*. Specific guidance is given for this important category of welds largely because of the significance of the different metallurgical zones in these weldments. Both defect-free and cracked conditions are considered. Creep rupture and continuum damage are assessed, as well as creep crack growth. V7 uses a much simpler methodology for estimating C(t) than V4/5. However, only steady loading is addressed, not cyclic conditions – and V7 is restricted to ferritics. A further limitation is that, in the creep crack growth part, only redistribution of secondary stresses is implicit, but not relaxation. Hence, if it is necessary to consider crack initiation, or fatigue, or cyclic effects on creep (creep-fatigue), or relaxation, then V2/3 and/or V4/5 must also be used, possibly in conjunction with V7. However, V7 is the workhorse behind much of the routine outage assessments of high temperature AGR piping systems (e.g., rupture lives). Volume 7 forms a large part of the subject matter of SQEP sub-role T73S06 (rupture parts), but also contributes to T73S03 (crack growth) and T72S04 (crack initiation).

Appendix A
Equation Set for Plane Stress Annular Plate under Equi-Biaxial Tension

Geometry: Flat circular plate of radius *b* with a concentric hole of radius *a*.

Loading: A radial tensile stress, σ_0, is applied around the whole of the outer circumference. For $b \to \infty$ this would be equi-biaxial tension remote from the central hole. The solution will also be developed for small *b*.



The geometry and loading are both axisymmetric, so there is no shear stress in (r, θ) coordinates, and all quantities are independent of θ. Consequently the general equations simplify considerably, as follows.

Equilibrium reduces to just one equation,

$$\frac{\partial \sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r} \qquad \text{(A.1)}$$

where both the hoop, σ_θ, and radial, σ_r, stresses are independent of θ. In plane stress, Hooke's Law is,

$$E\varepsilon_r^e = \sigma_r - \nu\sigma_\theta \qquad \text{and} \qquad E\varepsilon_\theta^e = \sigma_\theta - \nu\sigma_r \qquad \text{(A.2)}$$

Finally, we need compatibility. The strains are given in terms of the radial displacement for this axisymmetric problem by,

$$\varepsilon_r = \frac{\partial u_r}{\partial r} \qquad \text{and} \qquad \varepsilon_\theta = \frac{u_r}{r} \qquad \text{(A.3)}$$

noting that these are total (elastic + creep) strains. These give a simple form of compatibility relation for this problem,

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = \frac{\partial(r\varepsilon_\theta)}{\partial r} = \varepsilon_\theta + r\frac{\partial(\varepsilon_\theta)}{\partial r} \tag{A.4}$$

Subject to the boundary conditions $\sigma_r(r=a)=0$ and $\sigma_r(r=b)=\sigma_0$, Equs.(A.1), (A.2) and (A.4) suffice to solve the problem. Substitution of Equs.(A.2) into (A.4), and use of (A.1), yields,

$$\frac{\partial\sigma_\theta}{\partial r} = E\left(\frac{\varepsilon_r^c - \varepsilon_\theta^c}{r} - \frac{\partial\varepsilon_\theta^c}{\partial r}\right) - \frac{\sigma_\theta - \sigma_r}{r} \tag{A.5}$$

In the elastic case, (A.1) and (A.5) become $\frac{\partial\sigma_r}{\partial r} = -\frac{\partial\sigma_\theta}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r}$, which suffices to solve the elastic problem given the boundary conditions. The numerical solution algorithm used for the creep problem was,

[1] Define a grid of spatial points along a radius;

[2] Initial time to zero;

[3] Set some suitably small time increment;

[4] Evaluate the creep term, $E\left(\frac{\varepsilon_r^c - \varepsilon_\theta^c}{r} - \frac{\partial\varepsilon_\theta^c}{\partial r}\right)$, in (A.5) for all spatial points and for the current time, strains and stresses. (This will be zero for the first, elastic, step);

[5] Set radial stress at the hole to zero, $\sigma_r(r=a)=0$. The hoop stress at the hole initially retains its value from the previous step. On the first (elastic) step it is set to some arbitrary value, $\sigma_\theta(r=a)=\tilde{\sigma}$;

[6] Find the stresses at the next radial position using (A.1) for the radial stress and (A.5) for the hoop stress, noting that the creep term in (A.5) has been evaluated above;

[7] Factor all stresses by $\sigma_0 / \sigma(r=b)$;

[8] Redefine a suitable time increment (e.g., so that stresses change by a reasonable amount in the next step, say ~1 MPa);

[9] Repeat from [4] until some maximum required time is reached.

To evaluate the creep strains, some creep deformation law is needed. The program written to implement this solution procedure assumes time hardening for simplicity, and a power-law type behaviour. Thus the Mises strain rate is given by,

$$\dot{\bar{\varepsilon}}^c = C \cdot t^m \cdot \bar{\sigma}^n \tag{A.6}$$

Putting $m = 0$ permits secondary creep. For a Mises material, the normality rule gives the components of creep strain rate to be,

$$\dot{\varepsilon}_r^c = \frac{2\sigma_r - \sigma_\theta}{2\bar{\sigma}}\dot{\bar{\varepsilon}}^c \quad \text{and} \quad \dot{\varepsilon}_\theta^c = \frac{2\sigma_\theta - \sigma_r}{2\bar{\sigma}}\dot{\bar{\varepsilon}}^c \tag{A.7}$$

Where,

$$\bar{\sigma} = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r\sigma_\theta} \tag{A.8}$$

Note that the Mises strain is defined by,

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + (\varepsilon_\theta - \varepsilon_z)^2 + (\varepsilon_r - \varepsilon_z)^2} \tag{A.9}$$

The same definition of Mises strain is used here for elastic, creep and total strains. Finally, note that the z-strains are given by,

$$\varepsilon_z^c = -(\varepsilon_r^c + \varepsilon_\theta^c) \qquad \text{and} \qquad \varepsilon_z^e = -\nu(\sigma_r + \sigma_\theta)/E \tag{A.10}$$

A Poisson's ratio $\nu = 0.3$ has been used here. Equs.(A.6-10) permit the solution to be found for any material parameters C, m, n, E and applied stress σ_0.

Appendix B
Derivation of Tresca Reference Stress for Problem of Appendix A

The equilibrium equation in polar coordinates is,

$$r\frac{\partial\sigma_r}{\partial r} + (\sigma_r - \sigma_h) = 0 \tag{B.1}$$

where the subscripts denote the radial and hoop stresses. The nature of the problem is that both the radial and the hoop stresses are positive everywhere, and the hoop stress is larger than the radial stress (see graphs in main text). However, the out-of-plane stress is zero, since the problem is assumed to be plane stress. Consequently, assuming the yield stress is reached everywhere (the limit condition) gives simply $\sigma_h = \sigma_y$ assuming Tresca yield theory. Hence, Equ.(B.1) becomes simply,

$$r\frac{\partial\sigma_r}{\partial r} + \sigma_r = \sigma_y \tag{B.2}$$

But $r\frac{\partial\sigma_r}{\partial r} + \sigma_r \equiv \frac{\partial(r\sigma_r)}{\partial r}$. Hence, integrating and requiring that $\sigma_r = 0$ at $r = a$ gives,

$$\sigma_r = \left(1 - \frac{a}{r}\right)\sigma_y \tag{B.3}$$

But the applied load is $\sigma_r = \sigma_0$ at $r = b$, so we must have,

$$\sigma_0 = \left(1 - \frac{a}{b}\right)\sigma_y \tag{B.4}$$

This is the collapse solution. Since the average hoop stress is just $\langle\sigma_H\rangle = \frac{\sigma_0}{1 - a/b}$ we have simply that the collapse solution is just,

$$\langle\sigma_H\rangle = \frac{\sigma_0}{1 - a/b} = \sigma_y \tag{B.5}$$

The average hoop stress is therefore also the reference stress.