SQEP Tutorial T73S02: Session #14

last update 20/8/15

Handbook solutions for K; Qualitative behaviour of K for different geometries and loadings; Relative magnitudes of K for part-penetrating and through-thickness cracks; Controlling dimension for semi-elliptic cracks; Controlling dimension for self-equilibrating stresses and through-cracks; Bueckner's principle; Weight function concept (reference state method); Asymptotic K for deep cracks; Role of edge cracked plate in providing upper and lower bounds for cylinders; The effective SIF for through-cracks based on the average G; The SIF at the intersection of two perpendicular crack fronts: local and averaged; The SIF for a crack emanating from a notch radius; The SIF for a crack emanating from a sharp notch; Example shear mode SIFs.

WARNING: The SIF solutions presented here are illustrative only – DO NOT USE these notes as a source of SIFs for use in assessments – use the original references.

All SIFs herein are Mode I unless otherwise stated (for K read K_I).

Qu.: Why won't $K = \sigma_0 \sqrt{\pi a}$ do for everything?

$K = \sigma_0 \sqrt{\pi a}$ applies for an embedded crack of length 2a in an infinite plate subject to a remote normal tensile stress of σ_0. We may not be dealing with an embedded crack; we may not be dealing with uniform (membrane) loading; and we will certainly not be dealing with an infinite plate.

Qu.: What is the SIF for an edge crack of length 'a' in a semi-infinite plate?

Answer: $K = 1.12\sigma_0 \sqrt{\pi a}$. Why? To be honest I've never looked at the derivation of the 1.12 factor. But the physical reason for it is that the free boundary means that the crack can open a little more than for the embedded crack – hence a slightly larger SIF.

Qu.: What sources of Stress Intensity Factor (SIF) Solutions are there?

To list just a few…

- R6 Section IV.3
- R-CODE
- API 579
- SACC ("Swedish R6")
- Murakami
- SINTAP Compendium

…but there are lots more. And you can always "do it yourself", which will probably mean FEA.

Even if attention is restricted to linearised stress distributions and cylindrical geometries there are a lot of combinations of cases:-

- Extended cracks, semi-elliptic cracks, through-thickness cracks;
- Axial or circumferential orientation;
- Membrane stress, wall-bending stress, or global bending stress.
- Different Ri/Ro, different semi-elliptic aspect ratios, range of a/t.

So we won't try to cover everything.

(1) Edge-Cracked Plates (SENT = "Single Edge Notch Tension")

Qu.: What is meant by "bending restrained" and "bending unrestrained"?

Consider a long plate with a uniform membrane stress applied at its (distant) ends. If we introduce an edge-crack in the middle section of the plate, the opening of the crack will tend to make the plate bend. This is because the reaction load on the ligament is no longer symmetrical about the plate centre-line. Hence this reaction load is offset from the line of action of the applied load, which constitutes a bending moment. This is "bending unrestrained".

The alternative is to apply the remote loading in such a manner as to counteract this tendency to bend. Thus, when a crack is introduced, the remote stress is no longer uniform over the ends of the plate. Its distribution changes so that its mean line of action aligns with that of the reaction load on the ligament. This is "bending restrained"

Bending restrained and unrestrained are the extremes of a continuum of possible boundary conditions at the ends of the plate. The bending unrestrained case produces a much larger SIF (see Figure below).

(2)Extended Cracks in Cylinders

Qu.: Which has the larger *normalised* SIF (K/K_0), an axial or a circumferential crack?

For a given cylinder geometry, and a given extended crack depth, and assuming the same Mode I stress in each case (so that $K_0 = \sigma_0 \sqrt{\pi a}$ is the same), which crack orientation is the more onerous?

Answer: the axial K/K_0 is the larger.

Qu.: Why?

Because a cylinder with an axial crack can bulge more easily than one with a circumferential crack.

Qu.: For a cylinder under internal pressure, which has the larger absolute SIF – the axial or the circumferential crack?

The hoop stress is larger than the axial stress (by about a factor of 2 for thin cylinders). This leads to the value of K_0 for an axial crack being larger than that for a circumferential crack. Since the axial crack also has the larger K/K_0, the absolute SIF for the axial crack is clearly larger – generally by more than a factor of 2.

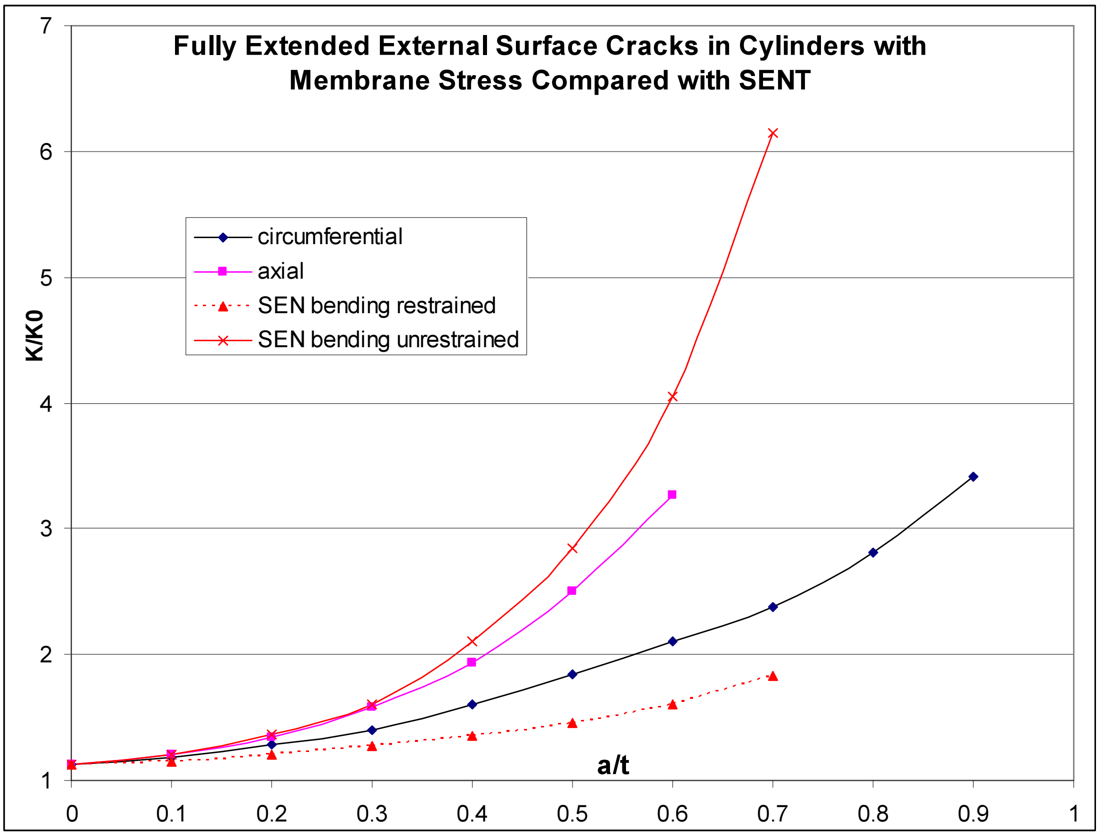
Qu.: How do the normalised SIFs (K/K_0) for cylinders compare with those for plates?

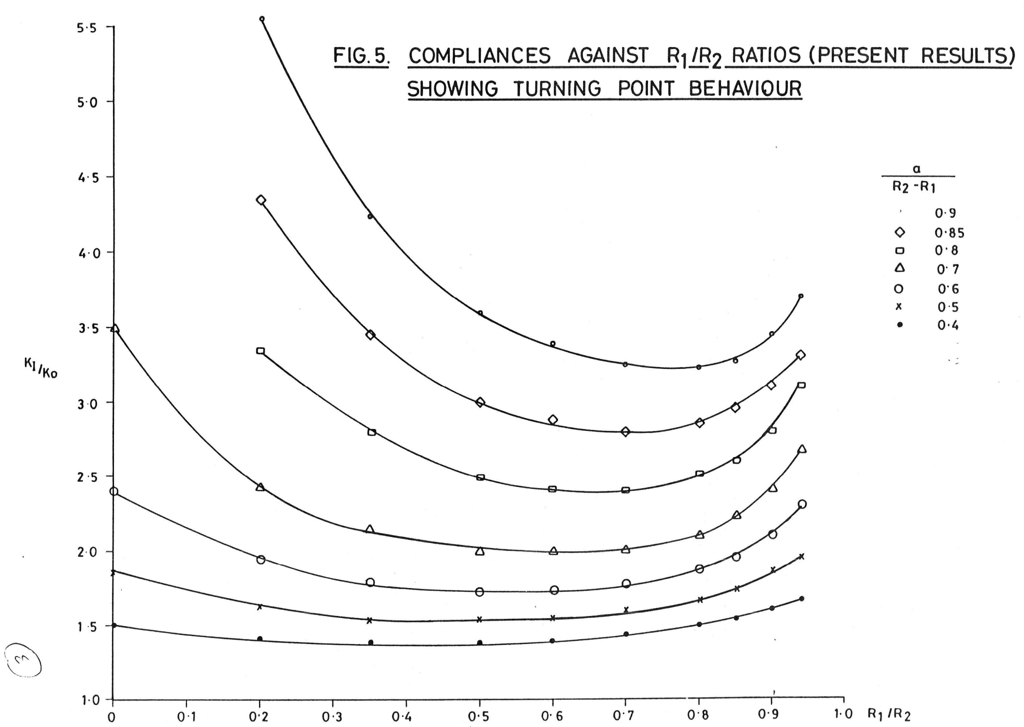
The bending restrained and unrestrained SENT cases act as lower and upper bounds respectively for the normalised SIFs of cylinders. So the order is,

$$K^{SENT}_{restrained} < K_{circumferential} < K_{axial} < K^{SENT}_{unrestrained}$$

Qu.: How do the SIFs vary with cylinder thickness parameter (Ri/Ro)?

The variation of the SIF with Ri/Ro can be surprising. For example, for fully circumferential cracks under membrane stressing, there is an Ri/Ro ratio which minimises the SIF for a given crack depth (see Figure).





External Fully Circumferential Cracks in Cylinders: Axial Membrane Stress

(Unfortunately the source of the above, due to Vissaridis, was permanently lost when the SAG General File was thrown away)

Qu.: Don't many SIF solutions apply only for a/t < 0.8? What to do for deeper cracks?

Yes, many SIF solutions are restricted to something like a/t < 0.8. For this reason it is common to see critical crack depths quoted in reports as equal to 80% of the wall thickness – with a note that the assessment was limited by the SIF solution.

Often it is not necessary to restrict yourself in this way because the behaviour of the SIF for deeper cracks can be estimated or bounded.

Qu.: What is the asymptotic behaviour of SIFs for deep cracks?

For plate geometries under applied tension or bending the SIF diverges as $a/t \to 1$. Call the normalised ligament $\xi = 1 - a/t$. The dominant singularity for sufficiently deep cracks is as follows,

SENT bending restrained: $K \to O\left(\frac{1}{\sqrt{\xi}}\right)$

SENT bending unrestrained: $K \to O\left(\frac{1}{\xi^{3/2}}\right)$

SENB pure bending: $K \to O\left(\frac{1}{\xi^{3/2}}\right)$

But we can do better than that. Algebraic expressions for K/K_0 can be derived from the asymptotic behaviour which are quite accurate for a/t greater than about 0.7 or so. This neatly complements the solutions which are available for smaller a/t. These expressions are available on http://rickbradford.co.uk and are illustrated in the Figures below.

Qu.: What about the asymptotic form for cracks in cylinders?

We have already observed that SENT restrained/unrestrained provide lower/upper bounds to the cylinder SIFs. So it follows that the asymptotic behaviour of the cylinder SIFs is bounded by $K \to O\left(\frac{1}{\sqrt{\xi}}\right)$ and $K \to O\left(\frac{1}{\xi^{3/2}}\right)$. Again we can do much better than this. By cunningly introducing the stiffness of the cylinder into the problem, the asymptotic methods which lead to the SENT solutions for deep cracks can be adopted for cylinders too. The wonderfully arcane expressions which result (originally due to Mike Heaton) can be found at http://rickbradford.co.uk/DeepCracks1.pdf and http://rickbradford.co.uk/DeepCracks2.pdf. They are illustrated in the Figures below.

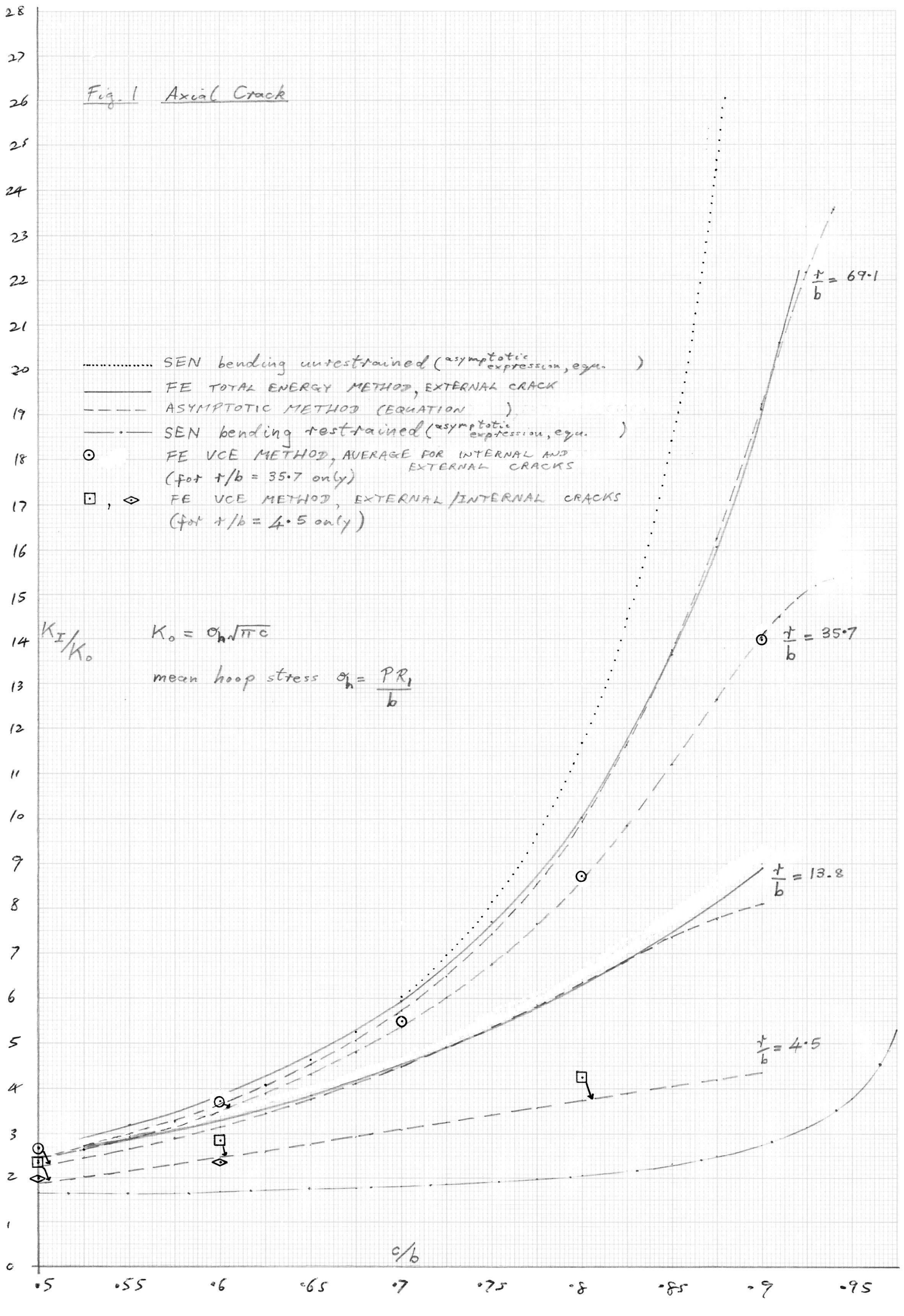
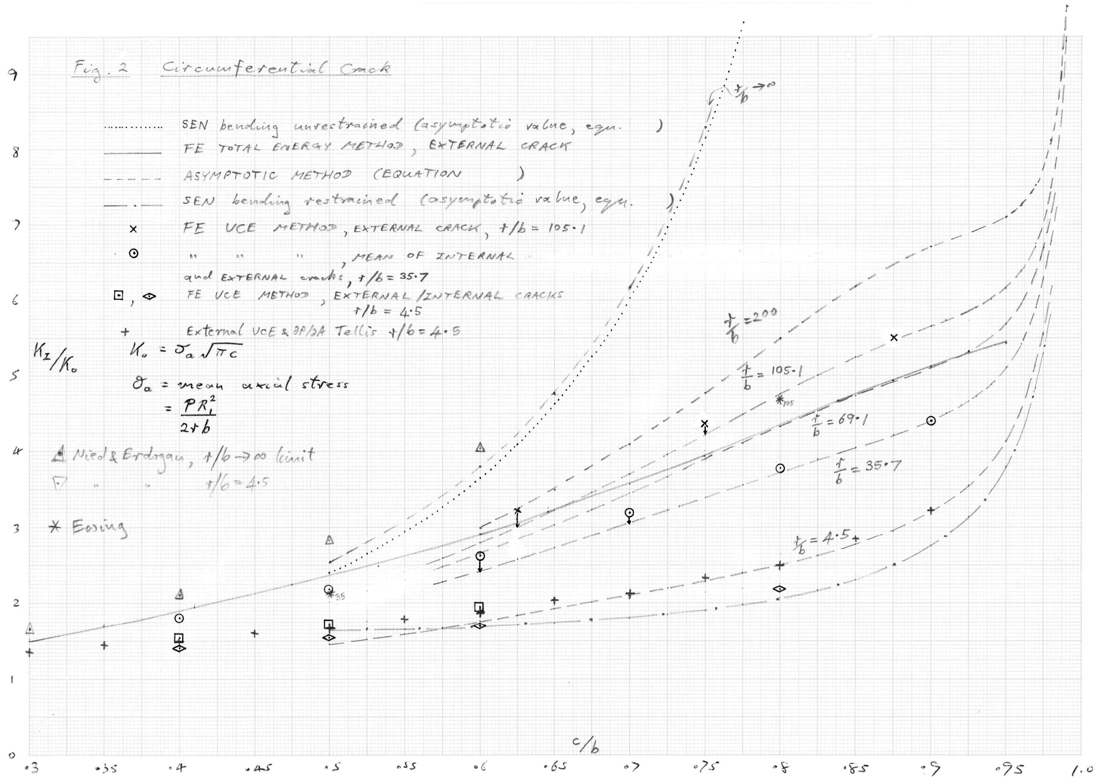


Fig. 1 Axial Crack
SEN bending unrestrained (asymptotic expression, equ.)
FE TOTAL ENERGY METHOD, EXTERNAL CRACK
ASYMPTOTIC METHOD (EQUATION)
SEN bending restrained (asymptotic expression, equ.)
FE VCE METHOD, AVERAGE FOR INTERNAL AND EXTERNAL CRACKS (for r/b = 35·7 only)
FE VCE METHOD, EXTERNAL/INTERNAL CRACKS (for r/b = 4·5 only)
K_I/K_o
$K_o = \sigma_h\sqrt{\pi c}$
mean hoop stress $\sigma_h = \frac{PR_1}{b}$
$\frac{r}{b} = 69{\cdot}1$
$\frac{r}{b} = 35{\cdot}7$
$\frac{r}{b} = 13.8$
$\frac{r}{b} = 4{\cdot}5$
c/b
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
·5 ·55 ·6 ·65 ·7 ·75 ·8 ·85 ·9 ·95



(3) Semi-Elliptic Circumferential Cracks in Cylinders (depth a, length 2c)

Qu.: Where is the SIF greatest – at the deepest point or at the surface?

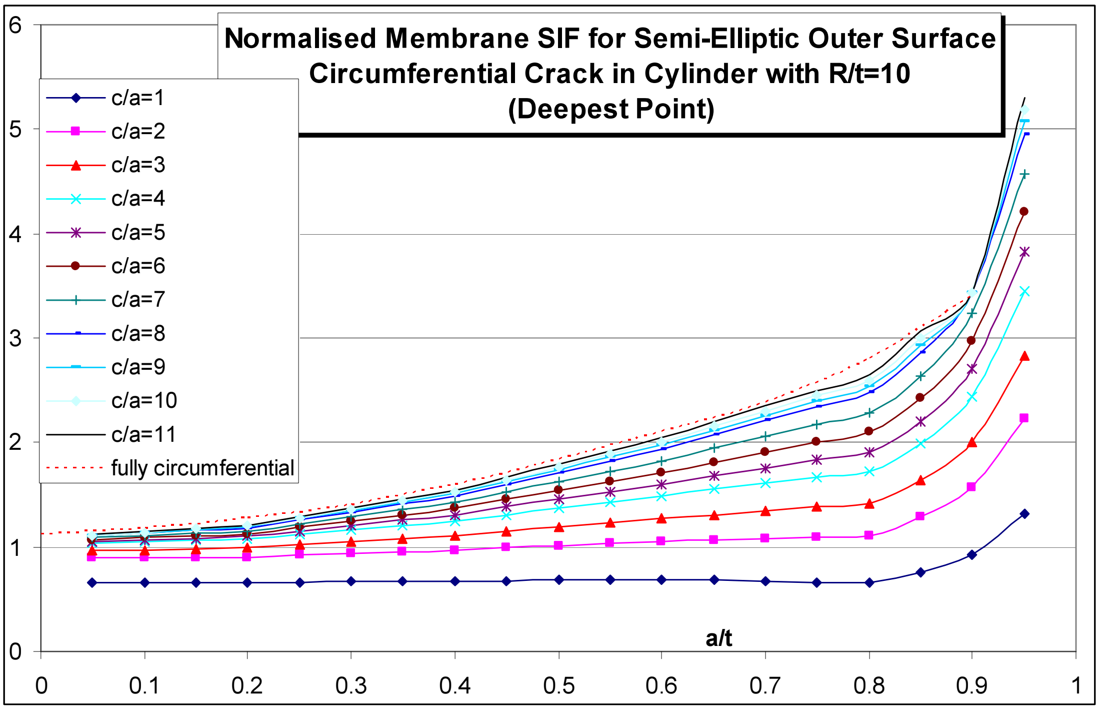
The only safe advice is to evaluate both & use the greater. However, if the stress is membrane then the deepest point will have the larger SIF as long as the crack is longer than about 3a (but for a semi-circular crack, the surface SIF is slightly larger). For a bending stress, however, it is not so clear because the SIF at the deepest point reduces as the crack becomes deeper – and so the surface point may have the larger SIF.

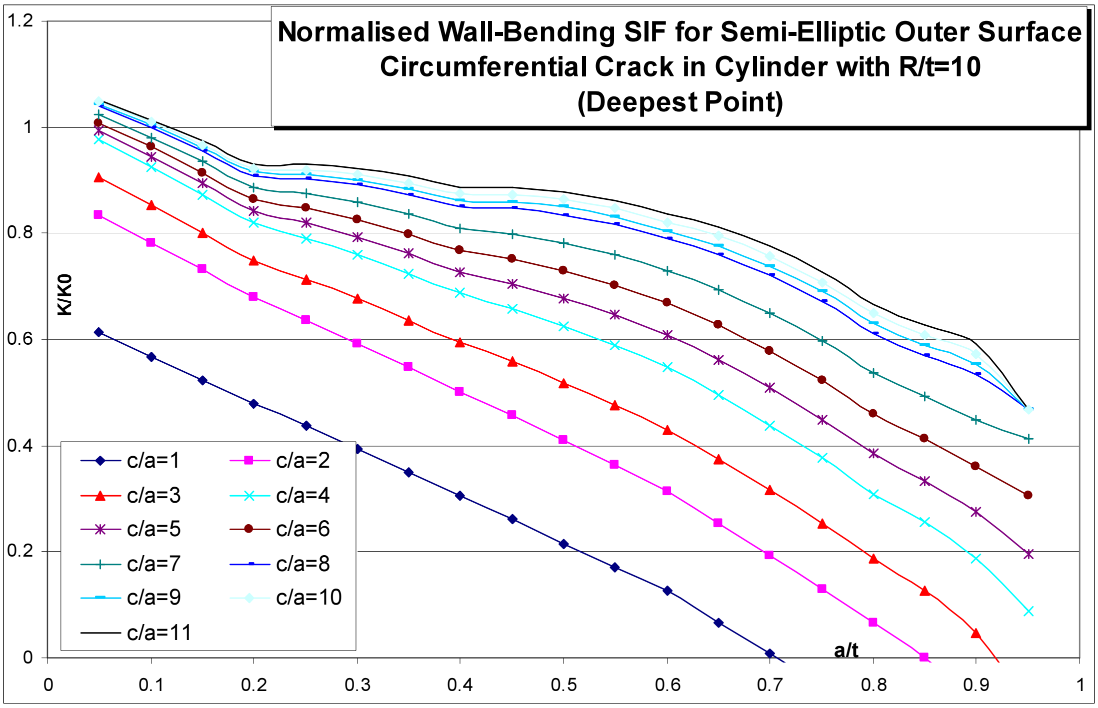
Qu.: What dimension controls the SIF magnitude (i.e., is used in K_0)?

Of course you could use either ‘a’ or ‘c’ to define K_0, but it is best to use the definition which makes K/K_0 closest to unity. This is the depth dimension, ‘a’. Hence it is conventional to define $K_0 = \sigma_0\sqrt{\pi a}$. Using this definition means that for long semi-elliptic cracks, K/K_0 tends towards the same value as for extended cracks.

Qu.: Is this mere convention, though?

No. There is also an important message. The crack length, 2c, may be 10 or 20 times larger than the depth, a (or more). But unless the crack is very deep, the largest SIF will be much closer to $\sigma_0\sqrt{\pi a}$ than to $\sigma_0\sqrt{\pi c}$. The latter will be a substantial over-estimate for long cracks with (say) $a/t < 0.6$, both at the deepest point and the surface point. In this sense it is the depth which controls the SIF, not the length.



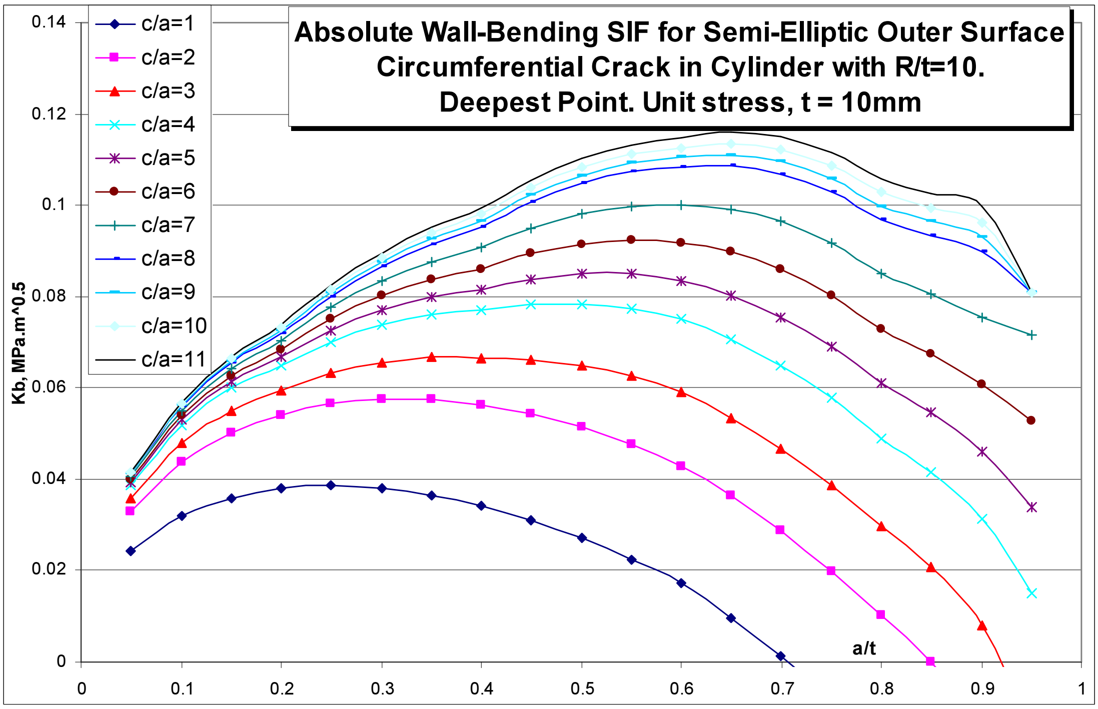


Qu.: What is the obvious qualitative difference in the plots of the normalised SIFs for membrane and wall-bending stress?

No prizes for this: K/K_0 increases for membrane stress but decreases for wall-bending stress.

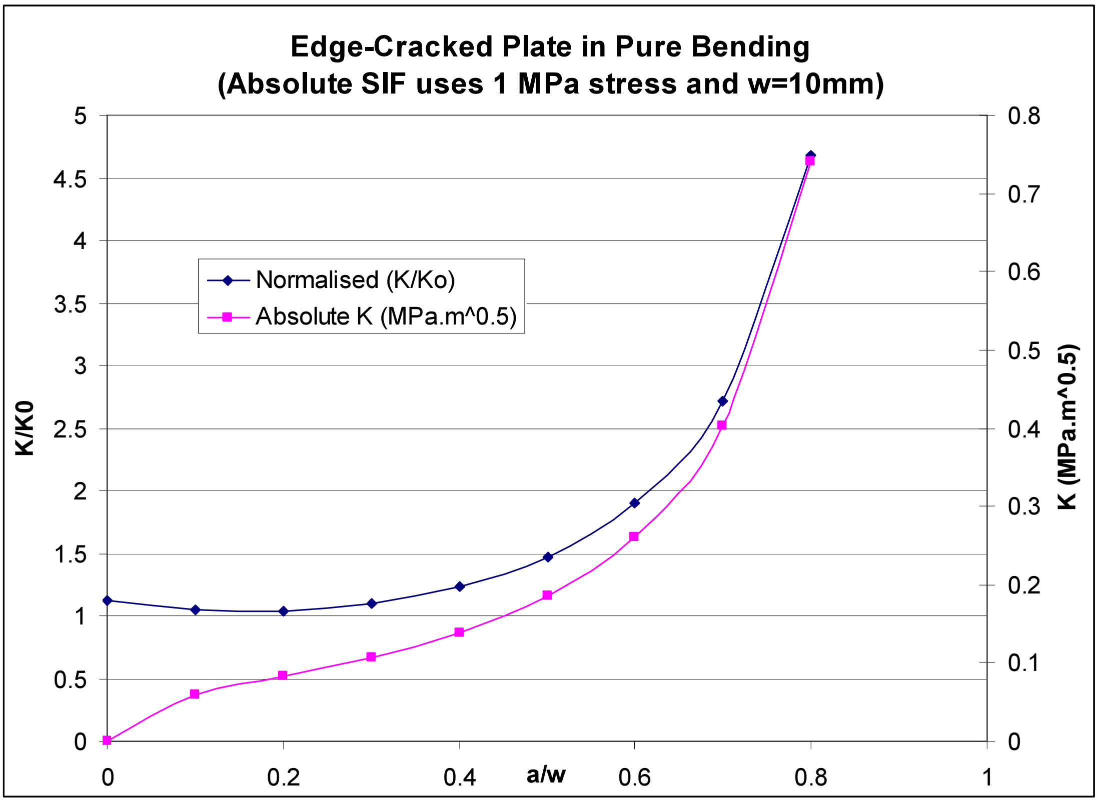
Qu.: So, the wall-bending SIF decreases as the crack gets deeper?

Careful! The absolute SIF equals the normalised value times $K_0 = \sigma_0 \sqrt{\pi a}$, so that $\sqrt{a}$ factor can lead to the SIF increasing despite K/K_0 decreasing. Actually, there is a turning point, and the absolute SIF reaches a maximum for some depth under wall bending (see Figure).



Qu. Why does the axisymmetric wall bending SIF reduce to zero as $a \to t$?

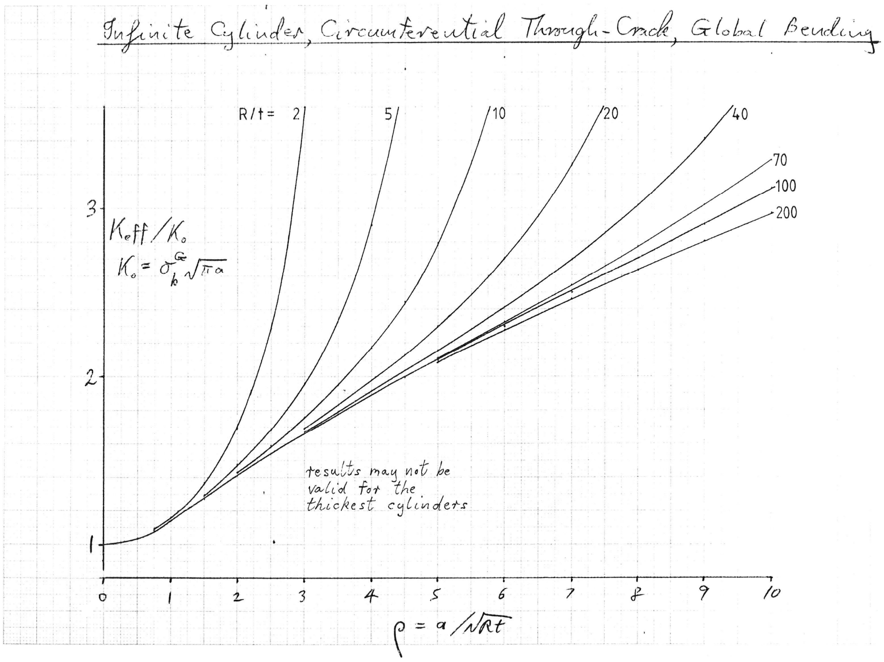
For a circumferential crack with wall bending the SIFs reduce to zero for deep cracks as a consequence of the stress being secondary. This must be so because an axisymmetric moment through the wall adds vectorially to zero, i.e., to zero net moment. This contrasts with the case for an edge cracked plate in pure bending – for which the moment is really there – and hence the SIFs increase monotonically…..

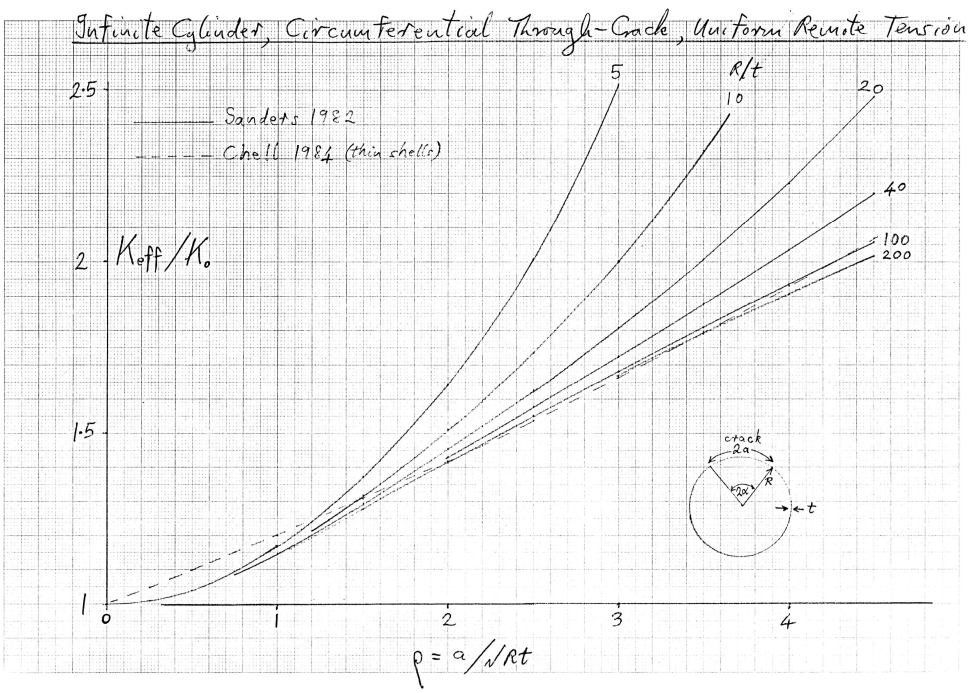


(4) Circumferential Through-Cracks

Qu.: If a semi-elliptic crack of length 2c and depth a ≈ half wall snaps through the wall thickness to become a through-crack of length 2c >> t, what happens to the SIF?

The SIF will generally become much larger. The point here is that the maximum SIF for the semi-elliptic crack is controlled by the depth dimension, so that $K_0 = \sigma_0 \sqrt{\pi a}$. In contrast, the SIF for the through-crack is controlled by the length, 2c, and hence $K_0 = \sigma_0 \sqrt{\pi c}$. Of course there are also the differing normalised SIF ('compliance') factors, K / K_0, to take into account. But assuming these are of similar magnitude, the fact that $c >> a$ leads to the through-crack having a much larger SIF. *In the two Figures below, 'a' is the half-length,*





Qu.: How does the SIF for a through-crack vary through the thickness?

The loading on the shell may have been described by a combination of tension, global bending and wall bending. SIF solutions for thin shells will probably provide a linearised SIF for each of these loadings, i.e., effectively a separate SIF on each of the inner and outer surfaces with a linear variation between the two. It follows that such SIF solutions imply that the SIF is greatest on one of the surfaces. R6 advises the use of the greater of the two surface SIFs in this situation.

However, for thick shells, or when the stress distribution is non-linear through the wall, the SIF could be maximum sub-surface. To ensure conservatism this maximum should be used in assessments.

Qu.: Are there alternative approaches?

Yes. For ductile materials it can be argued that the maximum elastic SIF along the crack front is too onerous. Instead some sort of weighted average of the SIF along the crack front might be more realistic. The obvious 'weighting' is to use the average of the energy release rate along the crack front. If the SIF is Mode I and its distribution is linear with membrane and bending components K_m and K_b, this leads to $E'\langle G\rangle = K_m^2 + \frac{1}{3}K_b^2$, and hence the effective SIF to use might be $K_{eff} = \sqrt{K_m^2 + \frac{1}{3}K_b^2}$. This is easily derived as $\frac{1}{t}\int_{-t/2}^{+t/2}\left(K_m + \frac{2x}{t}K_b\right)^2 dx$.

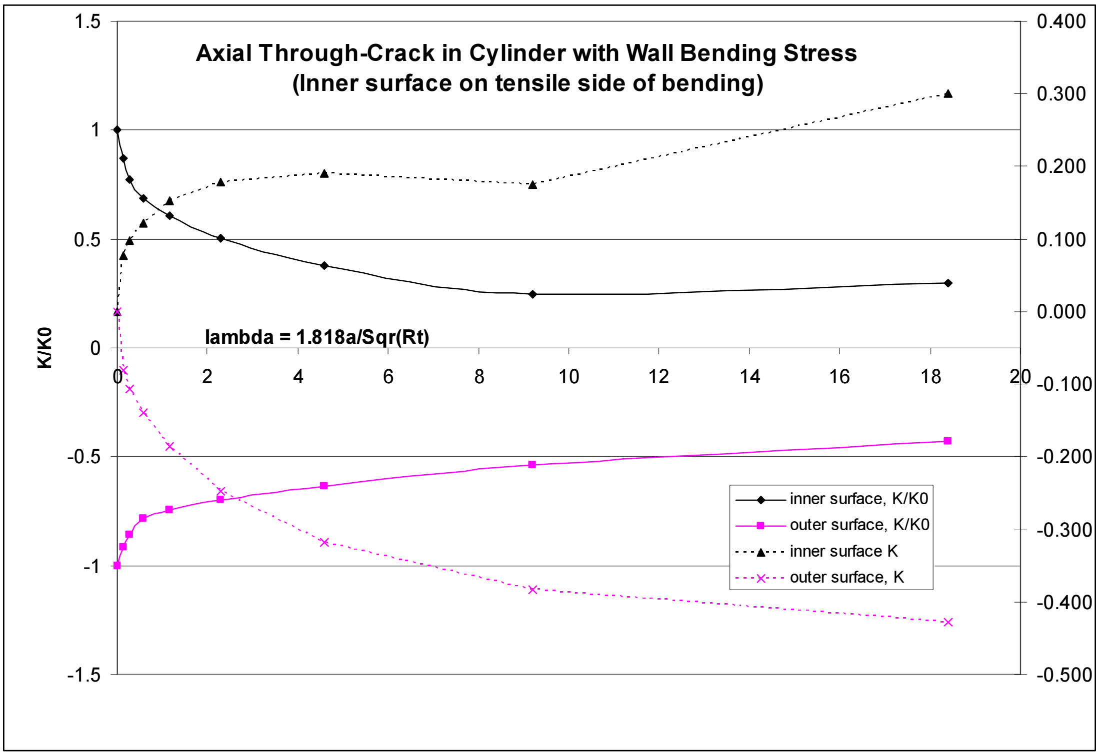
This method is mentioned within R6, but not sanctioned as such. Rather it is up to the user to justify its application on a case by case basis. A necessary requirement is sufficient ductility. In truth this approach is a means of sneaking a small amount of stable tearing into the assessment without explicitly saying so.

Qu.: What about the SIF for through-cracks with completely self-balancing stresses?

The stress distribution through the wall is said to be "completely self balancing" if there is zero net force and zero net moment. Residual stresses might be of this form, for example. R6 has special advice for such SIFs (see R6 II.6.7 and Figure II.6.3). These SIFs are not controlled by the crack length, but by the wall thickness, w, so that K_0 is defined as $\sigma_0\sqrt{\pi w}$, where σ_0 is the maximum of the non-linear stress distribution. Moreover, the normalised SIF, K/K_0, is always less than 0.5. Since the crack length, 2c, will generally be greater than the wall thickness, this means that the SIFs for such stress distributions are potentially relatively small. But not necessarily – because the residual stresses may be of yield magnitude.

Qu.: How do wall-bending SIFs behave for axial cracks?

The R6 solution for an axial through-crack (R6 Table IV.3.4.9.1) is plotted in the Figure below. The absolute SIFs apparently increase slowly as the crack gets longer. Unlike the case for circumferential cracks, a wall bending hoop stress does correspond to a net load, and hence the SIF would be expected to increase monotonically.



Qu.: If a crack emanates from a stress concentration, what is the SIF?

There are also solutions for cracks emanating from holes or notches.

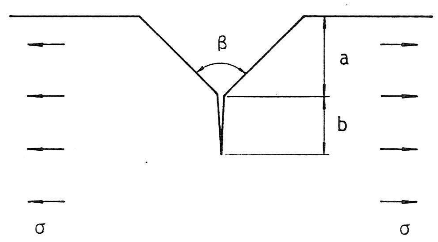
Crack from Notch

In the case of a notch on the edge of a plate, it is physically reasonable that the controlling dimension will be the sum of the notch depth (a) and the crack depth (b), so that the controlling K_0 is $\sigma_0\sqrt{\pi(a+b)}$. However, for small cracks ($b << a$) and notches of 'gentle' angle, this will be an over-estimate and K/K_0 will be substantially less than 1 because a better approximation is just $\sigma_0\sqrt{\pi b}$. This is illustrated below.

Crack from Hole

Consider a hole in an infinite plate subject to uniaxial tension, σ_0. In this case, if the crack length (a) is small compared with the radius of the hole (R), then the crack will effectively be exposed to 3 times the remote stress (because the SCF at the hole is 3). Hence, for a << R, $K \approx 3 \times 1.12\sigma_0\sqrt{\pi a}$ so that if we define $K_0 = \sigma_0\sqrt{\pi a}$ then $K/K_0 = 3.36$ for very small cracks. However, as the crack gets bigger, the mean stress acting over its length reduces. By the time that a = R, we might expect K to be roughly $\sigma_0\sqrt{\pi(R+a)} = \sqrt{2}\sigma_0\sqrt{\pi a}$ and hence K/K_0 to be $\sim\sqrt{2}$, and decreasing further towards unity as the crack gets deeper still. This is illustrated below.

3.15 A CRACK ORIGINATING FROM A TRIANGULAR NOTCH OF A SEMI-INFINITE PLANE UNDER UNIFORM TENSION



[Reference] N.Hasebe and J.Iida[1]
[Method] Conformal Mapping Function
[Accuracy] Less than 1%

$$K_I = F_I \sigma \sqrt{(a+b)\pi}$$

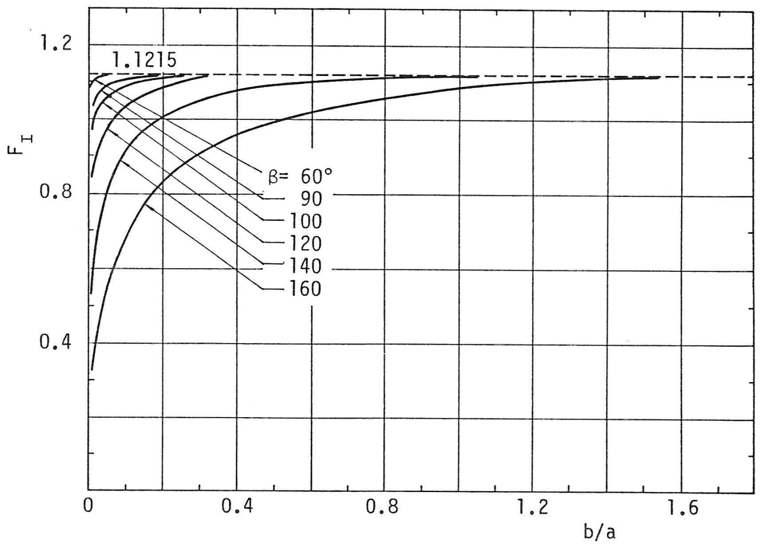
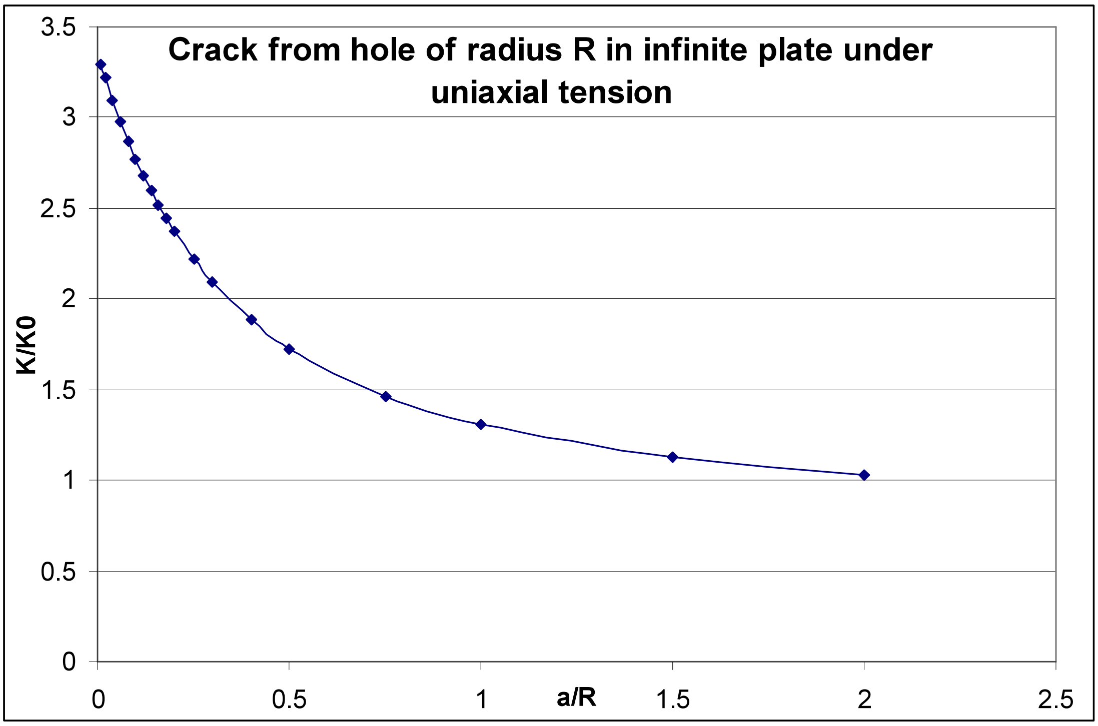


Fig. F_I values for b/a



Qu.: What about shear mode SIF solutions?

In pure Mode III, and for a finite width plate, there is an analytic solution,

$$\frac{K_{III}}{K_{III0}} = \left[\frac{2b}{\pi a}\tan\left(\frac{\pi a}{2b}\right)\right]^{1/2}$$

where b is the plate thickness in the direction of the crack.

In Mode II, FE solutions have shown that as long as the plate width, h (perpendicular to the crack) is about double the plate thickness, b (in the direction of the crack), then this Mode III solution is a good representation of the Mode II SIF also – see Figure below.

FE analyses of Mode III which I carried out in 1981 agree with the above analytical solution, but only when the out-of-plane length, t, of the block of material modelled was large compared with the in-plane dimensions, b and h. When t was taken as comparable to b and h, then K_{III}/K_{III0} was much larger (see Figure). However, I'd treat this with great caution – I might have screwed up. I include it mostly because there is a dearth of shear mode solutions.

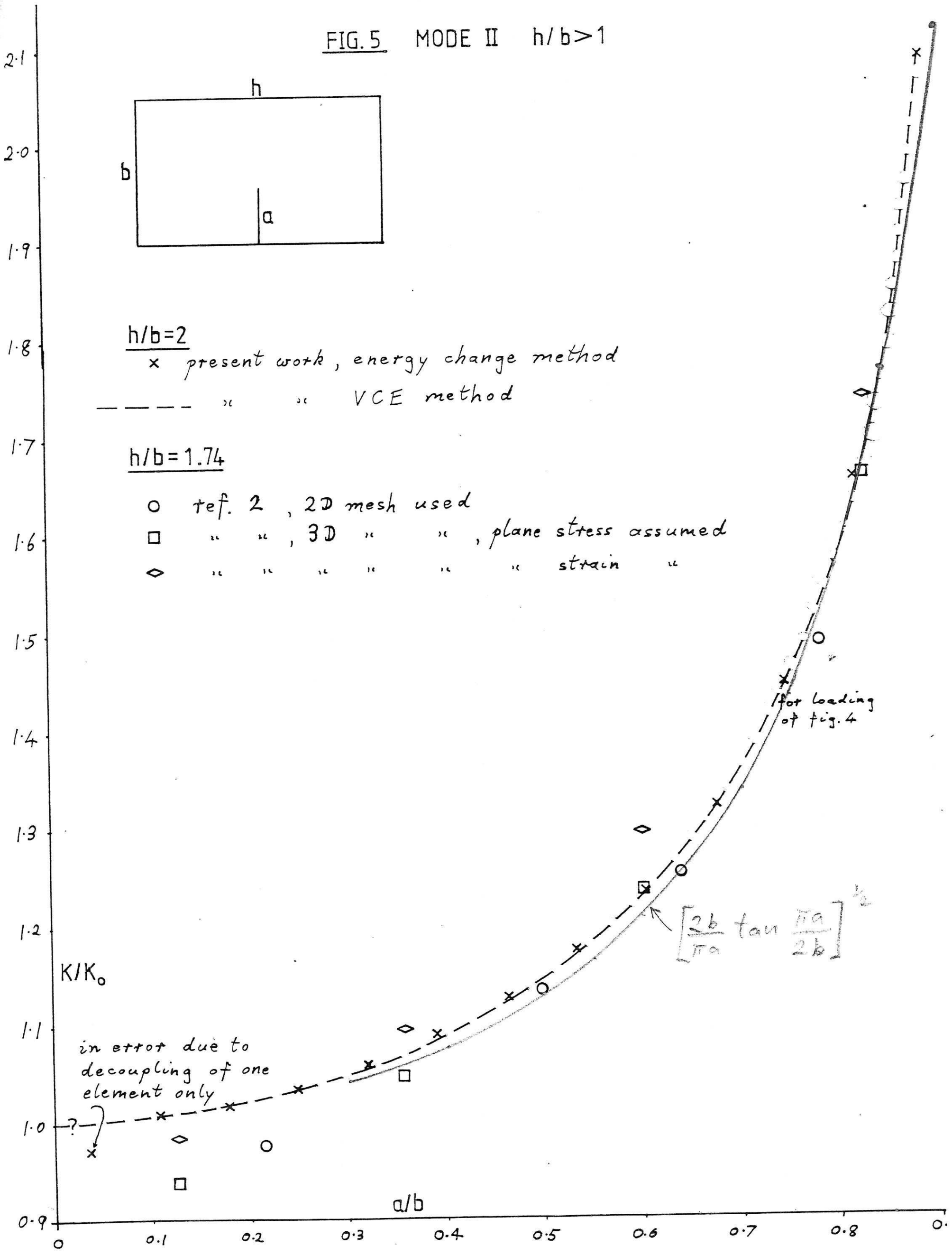
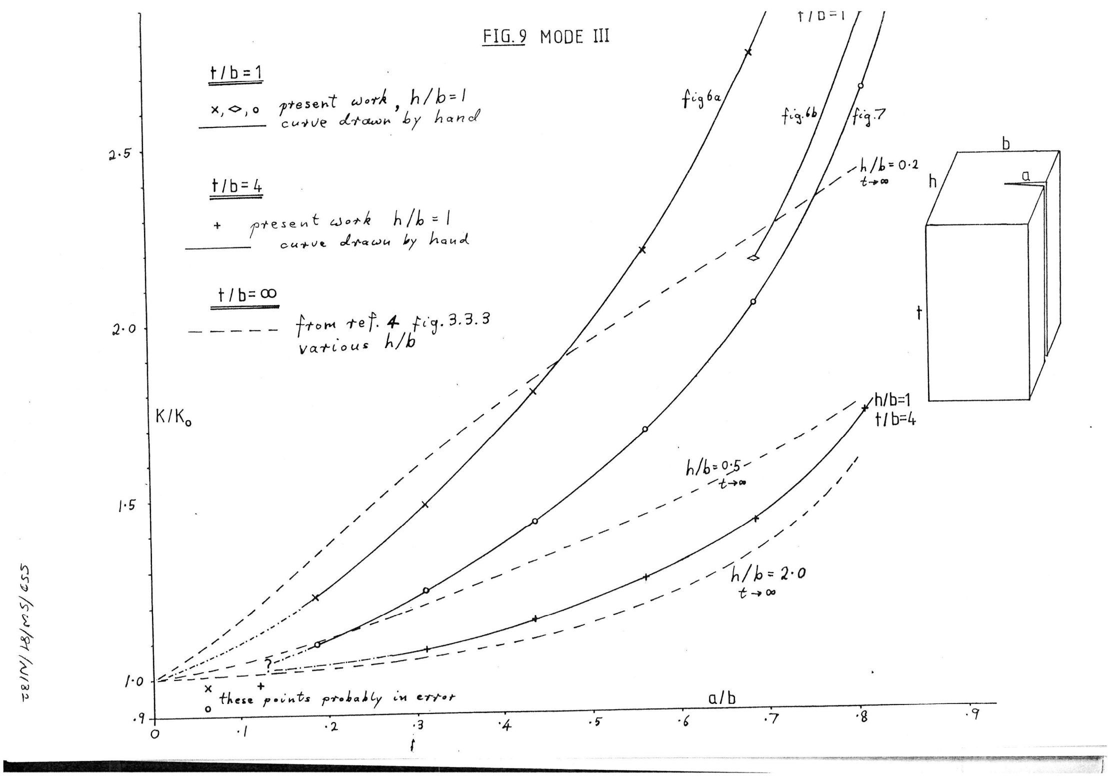


FIG. 5 MODE II h/b>1

552/SW/81/N182



Qu.: What about two intersecting cracks?

You may need to consider, say, a through-thickness crack in a weld which also has a fully circumferential crack. The latter might be an unfused land, for example. The elastic solution for intersecting crack fronts tends to produce a divergent K at the intersection point. This is because one crack experiences the divergent stress field of the other. (However, I think these divergences are logarithmic, i.e., relatively tame). This means that re-entrant corners on a crack front tend to get smoothed out by crack growth. If you really are dealing with a brittle material, then such irregularities on the crack front may reduce the load carrying capacity.

However, for ductile materials it may be that some weighted average K along the crack front is more relevant. In this case it is worth noting that it would not be correct to argue as follows: Say there is a through-crack of length *2c* in a wall of thickness *t* which also contains an extended part-penetrating crack of depth *a*. The part-penetrating crack increases the local ligament stress by a factor $1/(1-a/t)$, so the SIF for the through-crack in the presence of the part-penetrating crack can be estimated to be about $K_{TT}(c,a) = \frac{K_{TT}(c,0)}{1-a/t}$, where $K_{TT}(c,0)$ is what the through-crack SIF would be in the absence of the part penetrating crack.

This is wrong in general. I cannot give a reliable solution for all circumstances. However, so long as *a/t* is small, and as long as we interpret $K_{TT}(c,a)$ as the effective SIF derived by averaging *G* along the crack front, then $K_{TT}(c,a) \approx \frac{K_{TT}(c,0)}{\sqrt{1-a/t}}$. The reason for this is that an increase in the crack (half) length, δc, corresponds to a crack area increase (per crack tip) of $\delta A = (t-a)\delta c$, rather than $\delta A = t\delta c$ which would apply without the part-penetrating crack. Consequently it is the average *G*, rather than *K*, which is increased by the factor of $1/(1-a/t)$, and hence K increases only by a factor of $1/\sqrt{1-a/t}$. This ceases to hold for larger *a/t* because the part-penetrating defect starts to influence the energy release δP for a given δc.

Qu.: What is Bueckner's Principle?

Bueckner's Principle states that the same K results if the load acting on a structure is replaced by the following:-

(a) Remove all applied loads and displacements, and instead apply tractions to the crack faces equal & opposite to the stresses which would occur on the crack line in the uncracked body;

(b) All degrees of freedom which have prescribed displacements (zero or non-zero) are replaced by constraints (i.e., constrained to zero).

Bueckner's Principle applies only for linear elastic behaviour.

The original proof of Bueckner's Principle used an energy argument and applied only for primary loads. Mike Heaton later extended this energy-based argument to secondary stresses. However, the Principle is actually a simple consequence of linear superposition and is very simple to prove without such complicated arguments.

Proof: Consider the structure under its true loading conditions and with a crack which has been opened by the resulting stresses. By "opened" we include shear displacements of the crack faces. We now notionally apply tractions to the crack faces so as to bring them precisely back into coincidence, as they would be if the body were uncracked. It follows that these tractions are equal to the stresses in the uncracked body. Now these tractions have caused the crack face displacements to reduce from their true value under load, $\bar{u}(\bar{r})$, to zero. So, if we applied these tractions with reversed sign, they would produce crack face displacements of $\bar{u}(\bar{r})$, the same as the displacements due to the true loading. This follows because we are dealing with linear elasticity, and hence linear superposition of solutions applies. Note that this is true only if the prescribed degrees of freedom under the original loading are held fixed whilst the crack faces are loaded, since this would be the case during the notational crack closure.

But the LEFM K is uniquely defined by the displacements of the crack faces, and hence if the crack faces displace the same, then K must be the same. In conclusion, under conditions (a) and (b), above, we get the same K. QED.

Qu.: What are "weight functions"?

Generally handbook SIF solutions apply to linearised stress distributions (membrane stresses and bending stresses). Weight functions are a means of evaluating SIFs for arbitrary stress distributions. This can be valuable if the stress variation is markedly non-linear in the vicinity of a crack, so that linearization would be quite inaccurate.

Suppose we apply a stress $\sigma(x)$ to the crack faces between locations x and x+dx (applying an equal and opposite stress to the two crack faces). For a 2D case, and considering unit thickness, the corresponding force is $\sigma(x)dx$. The resulting SIF, dK, must be proportional to the applied load, so there must exist a function $W(a,x)$ such that,

$$dK = W(a,x)\sigma(x)dx$$

The notation recognises that the resulting SIF depends both upon the crack size, *a*, and upon the position, *x*, along the crack face at which the load is applied. For a stress distribution along the whole of the crack faces, integration gives the total SIF simply as,

$$K = \int_{crack} W(a,x)\sigma(x)dx$$

But, thanks to Bueckner's Principle, we know that the elastic SIF due to an arbitrary state of stressing equals the above integral – where we now interpret the stress $\sigma(x)$ as the stress in the uncracked body at that position. Consequently, if we know the weight function, $W(a,x)$, we will have the SIF solution for all possible stress states. The weight function depends upon the geometry of the body and the geometry of the crack – but not on the loading.

The weight function is known in closed form for a few simple cases. Examples are:-

Embedded Crack in an Infinite Plate:

$W(a,x) = \frac{1}{\sqrt{\pi a}}\left(\frac{a+x}{a-x}\right)^{1/2}$, so that the SIF for such a crack in an arbitrary stress field is

$K(a) = \frac{1}{\sqrt{\pi a}}\int_{-a}^{+a}\left(\frac{a+x}{a-x}\right)^{1/2}\sigma(x)dx$. (Crack tips are at $x = \pm a$)

Semi-Infinite Crack in an Infinite Plate: 3D Case where stresses vary in x and y directions

We now have to integrate over the area *dxdy* of the crack. The solution is,

$$K = \frac{1}{\pi}\int_{y=-\infty}^{+\infty}\int_{x=0}^{+\infty}\left(\frac{2x}{\pi}\right)^{1/2}\frac{\sigma(x,y)}{x^2+y^2}dxdy \qquad \text{(Crack tip is at } x=0\text{)}$$

Semi-Infinite Crack in an Infinite Plate: Case where stresses vary in x only

The y-integral in the above expression can readily be carried out when the stress is constant in the y-direction. This gives,

$$K = \sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{\sigma(x)}{\sqrt{x}}dx \qquad \text{(Crack tip is at } x=0\text{)}$$

The characteristic of all these solutions is that the stresses near the crack tip are weighted more strongly in the integral than stresses further away. In fact, W is singular at the crack tip, with a $1/\sqrt{r}$ divergence.

Qu. What is the R-CODE Reference State Method?

The R-CODE Reference State Method is an approximate method for finding the SIF due to an arbitrary stress distribution given the SIF solution for some "reference state", that is, for some other specific stress distribution. This is done by approximating the weight function as,

$$W(a,x) \approx \frac{2(1+mz)}{\sqrt{\pi a(1-z^2)}}$$

for some constant 'm', where $z = x/a$ and the crack tip is at $z=1$. This algebraic form captures the correct singular behaviour. Ignoring the *mz* term, all exact weight function solutions reduce to this expression sufficiently near the crack tip. The term in *mz* is what distinguishes one geometry from another.

The SIF for an arbitrary stress distribution is then,

$$K \approx 2\sqrt{\frac{a}{\pi}}\int_0^1\frac{\sigma(z)}{\sqrt{1-z^2}}(1+mz)dz$$

(This is for an edge crack. For an embedded crack the lower limit would be -1). Suppose we know the SIF for a given crack length and a given stress distribution (e.g. a membrane stress). This equation can then be solved for the constant '*m*'. The equation can then be used to find the SIF for an arbitrary stress distribution and the same crack length. In general, '*m*' may vary with crack length.

Note that the reference state method works only because a linear approximation for the numerator of W has been assumed. It is not possible to derive exactly the SIF for an arbitrary stress distribution from a single example solution, even if the latter is known for all crack lengths. Even if we knew *K(a)* for all '*a*' for a give stress distribution, $\sigma(x)$, it is not possible to solve the integral equation $K(a) = \int_{crack} W(a,x)\sigma(x)dx$ for $W(a,x)$. It becomes possible only by virtue of the linear approximation, which means that there is just a single variable (m) to be found. (I mention this only because it caused me great confusion at one time. R-CODE does not explain how the reference state solution works, and it appeared to me to be impossible. It is – unless the above approximation is introduced).

In practice you do not need to understand the above theory in order to apply the reference state solution in R-CODE. It is simple and automatic to use, and very useful.