Homework for T72S01 Tutorial #5: Equivalent Stresses & Mohr's Circles

Mentor Guide Knowledge & Skills Questions

1.7 State or derive the algebraic relationships between stress and strain for a linear elastic material, the definition of the elastic moduli and the relationship between the three elastic moduli for an isotropic material.

1.8 Define what is meant by: the principal stresses, the hydrostatic stress, the deviatoric stresses, the Tresca equivalent stress and the Mises equivalent stress.

Numerical/Algebraic Questions

1) Find the Mises equivalent stress, the Tresca equivalent stress, the hydrostatic stress, and the deviatoric stress matrix of,

$$\begin{pmatrix} 30 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 70 \end{pmatrix}$$

2) Find the Mises equivalent stress, the hydrostatic stress, and the deviatoric stress matrix of,

$$\begin{pmatrix} 30 & 10 & 20 \\ 10 & 40 & 30 \\ 20 & 30 & 70 \end{pmatrix}$$

Why is it harder to find the Tresca equivalent stress?

3) A stress state is defined by its three principal stresses, which are 10, 20 and 40 MPa. Use the 3D Mohr's circle construction to show that there is no orientation of an element of area such that the direct and shear stresses acting on the area are 16.2 and 2.2 MPa respectively. Conversely, show that it ***is*** possible to orient an element of area so that the direct and shear stresses are 18.7 and 6.5 MPa respectively.

4) Prove that the Mises stress is given in terms of the deviatoric stresses by $\bar{\sigma} = \left[\frac{3}{2}\hat{\sigma}_{ij}\hat{\sigma}_{ij}\right]^{1/2}$ given the usual definition,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)\right]^{1/2}$$

5) The state of stress at a point is given by the following stress matrix with respect to the (x,y,z) coordinate system:

$$\begin{pmatrix} 100 & 50 & 70 \\ 50 & 150 & 90 \\ 70 & 90 & 200 \end{pmatrix} \text{ MPa}$$

What is the direct (tensile) stress in the direction given by direction cosines $(0.7 \quad 0.5 \quad 0.51)$?

Plant Example:-

6) Whilst yielding (plastic strains) depend upon the equivalent stress, many failure mechanisms are sensitive to the hydrostatic stress, e.g., fracture and reheat cracking.

Reheat cracking is a creep mechanism in which the damaging stresses are primarily welding residual stresses. The cause of the cracking is high levels of hydrostatic stress, which reduces the creep ductility. The reason why welding residual stresses in thick sections promote reheat cracking is that they tend to produce large hydrostatic stresses. In simple qualitative terms, why should this be so? (One sentence will suffice – don’t go overboard on this one).