Chapter 9B – Fine Tuned Strong Force?
(3)The Binding of the Deuteron

1. Introduction

The strong nuclear force is poorly understood. The absence of an easily soluble theory of the strong force prevents an examination of its fine tuning which is both elementary and rigorous. Nevertheless, quantitative predictions are possible to a sufficient level of accuracy to display the approximate degree of fine tuning, as we have seen in Chapters 8 and 9. In Chapter 9 we considered the constraints on the strong nuclear and electromagnetic forces implied by the stability of the key biological elements. In this Chapter we consider a particular special case in more detail. This is the stability of the deuteron, the bound state of a proton and a neutron. Whilst the deuteron itself is not important for life, its significance lies in the fact that the formation of all the elements beyond hydrogen proceeds via deuterium. If the deuteron were not stable – at least for a few seconds – the chemical elements would not exist in the universe.

The strong nuclear force acts between any pair of nucleons which are sufficiently close. What is noteworthy is that, whilst a neutron and a proton can form a bound state, two neutrons or two protons cannot. In this Chapter we explain why this is the case. We shall examine the degree of fine tuning of the strong nuclear force required to bring about this state of affairs. How much weaker would the nuclear force have to be to result in no deuteron existing? How much stronger would the nuclear force have to be to result in bound dineutrons and diprotons? These questions are addressed in Section 2.

In Section 3 we make some preliminary remarks regarding the implications for the universe if these conditions were realised. The particular case of diproton stability is considered in greater detail in Chapter 9c. The reason for this special attention is that the implications of diproton stability have been repeatedly misrepresented in the literature. It has been claimed that diproton stability would result either in an all-helium primordial universe, or in a universe in which stars would explode. An all-helium primordial universe, being without hydrogen, and hence without water, hydrocarbons, hydrogen bonds, etc, would indeed be catastrophic. Similarly, the absence of stable stars would be catastrophic to elemental synthesis. However, diproton stability results in neither an all-helium universe nor in explosive stars, as will be demonstrated in detail in Chapter 9c.

The issue of diprotons seems to have attracted elementary blunders in the popular literature. Another example is in respect of the reason for diprotons not being bound in this universe. Gribbin and Rees in "Cosmic Coincidences" claim that, "In our universe, the electrical force of repulsion between two positively charged protons overwhelms the nuclear force of attraction between them, and diprotons do not exist". As we shall see below, the Coulomb repulsion between two protons is <u>not</u> the reason why they fail to bind together into a diproton. If it were, the dineutron would still exist, which it does not.

2. The Binding Of Two Nucleons

An elementary approach to the inter-nucleon nuclear force is to represent it as a potential and apply the Schrodinger equation. This approach is particularly well suited to bound-state problems. The magnitude and range of this nuclear potential is something which must be determined empirically. The strong nuclear force turns out to be spin dependent. Its strength is greater when the nucleon spins are aligned (that is, when they are in a spin triplet state, S = 1). When the spins are anti-aligned (i.e. in a spin singlet state, S = 0) the nuclear potential is less strong. It is not necessary, for the arguments that follow, to understand why there should be this spin dependence, though we might well wish to understand it better[1].

2.1 The Square Well Approximation

The simplest nuclear potential to assume is a 'square' well potential. Within a sphere of radius 'a' the potential is constant (and negative, of course) of magnitude V_0, whereas outside this sphere the potential is zero. The singlet and triplet states are ascribed different square-well potentials. Kaplan gives estimates for the nuclear potential based on np scattering experiments:-

state	**a (fm)**	**V_0 (MeV)**	**a^2V_0 (MeV.fm^2)**
singlet	2.8 (2.4)	11.8 (16.1)	92.5 (92.7)
triplet	2.054 (1.70)	35.1 (51.2)	148 (148)

The figures in brackets are an alternative interpretation of the experimental data. Kaplan also presents data to support the pp potential being the same as the np singlet potential.

The solution of the Schrodinger equation for a square well potential (Schiff Equ.15.3) shows that there is exactly one bound state if $a^2V_0M_R/\hbar^2$ lies between $\pi^2/8$ and $\pi^2/2$ where M_R is the reduced mass. (NB: Exceeding $\pi^2/2$ results in the first L = 1 bound state). We shall take M_R to be half the proton mass for sake of argument (i.e. ignoring the small neutron-proton mass difference).

Using the data in the above Table, the triplet state has $a^2V_0M_R/\hbar^2 = 1.779$. Since this lies between $\pi^2/8 = 1.234$ and $\pi^2/2 = 4.935$, it follows that the triplet spin state has exactly one bound state. By the exclusion principle, nn and pp cannot be in a triplet spin state and the same orbital state, so this conclusion applies only to np. Hence, the deuteron is "predicted" to be stable, but to have no excited states – which is correct. [The word "predicted" is inaccurate since the binding energy of the deuteron has been used, amongst other things, to determine the above nuclear potential strength].

Using the l = 0 energy level for a square-well potential given by Schiff, Equ.(15.3), we do indeed find that with a triplet potential given by a = 2.054 fm and V_0 = 35.08 MeV there is a unique bound state at E = -2.22 MeV, i.e. this potential reproduces the

[1] We note in passing that it might be related to the explanation of strong forces in terms of meson exchanges. Thus, the exchange of a (spinless) pion will contribute to the force between a pair of nucleons in either the singlet or the triplet state, whereas the exchange of a spin one meson (e.g. ρ) can contribute only to the singlet state (because of the spin flips that occur in the Feynman diagram, at least if we confine attention to the orbital L = 0 state). The extra term contributing to the singlet state would make the singlet force weaker if the sign of the rho exchange term were opposite that of the pion exchange. Alternatively, the inter-nucleon force should, in principle, be derivable from quantum chromodynamic field theory. Whether this has been done, and in particular whether the greater strength of the triplet force has been explained thereby, I do not know.

correct deuteron binding energy. We shall examine the square well energy levels in more detail below.

We see from the above figures that reducing the strong nuclear potential well depth, V_0, by just ~30% would result in the deuteron being unbound (that is, 0.7 x 1.779 ~ 1.234). The nuclear potential strength, V_0, may be related to the strong nuclear coupling strength.g_s, by $V_s(r) = \alpha_s f(r)$, where $f(r)$ is a step function $\frac{\hbar c}{a}\Theta(a-r)$ for a square-well potential, or $f(r) = \frac{\hbar c e^{-M_\pi r'}}{r'}$ in the case of a Yukawa potential, and where the nuclear force "structure constant" is $\alpha_s = \frac{g_s^2}{4\pi}\cdot\left(\frac{M_\pi}{M_N}\right)^2$. So a reduction of α_S by 30%, or alternatively a reduction of g_s by 16%, results in the deuteron being unbound.

In contrast, for the singlet state, $a^2V_0M_R/\hbar^2 = 1.113$. Since this is less than $\pi^2/8 = 1.234$ it follows that there is no singlet spin state which is bound. Thus, nn and pp are not bound. The dineutron would be stable if the strength of the strong force (as measured by V_0 or α_S) were increased by ~11% (i.e. if g_s were increased by ~5%). Hence, the dineutron is a remarkably close miss as regards stability.

To evaluate the increase in the strong force required to cause the diproton to become stable we must also account for the destabilising effects of the Coulomb repulsion. At the edge of the strong nuclear potential well (2.4fm to 2.8fm) the Coulomb energy is 0.51-0.60 MeV. Decreasing the effectively total potential well depth by this amount leads to $a^2V_0M_R/\hbar^2 = 1.065$ to 1.072. Increasing the nuclear force (as measured by V_0 or α_S) by 15%-16% (or ~8% in g_s) would therefore result in diproton stability. Again, diproton stability is a close miss.

Note that the dineutron and diproton could be stabilised by an increase in the strong force which is small compared with the increase that would be needed to give rise to a excited state of the deuteron (which is an increase by a factor of 4.935/1.779 = 2.77). Barrow & Tipler (P.322) quote results which are very similar to those we have derived above (see the Table in next Section).

2.2 The Yukawa Potential Approximation

In Chapter 13 an alternative approximation to the strong nuclear potential was discussed, namely the assumption of a Yukawa form rather than a square well. Since the singlet and triplet potentials are different, they correspond to different Yukawa terms. From the point of view of exchange particles, they would correspond to different exchange mesons and hence have different decay lengths. Thus we define 'central' and 'tensor' Yukawa potentials by,

$$V_c = V_{c0}\frac{e^{-r/a_c}}{r/a_c} \quad \text{and} \quad V_t = V_{t0}\frac{e^{-r/a_t}}{r/a_t} \tag{1}$$

Where Evans, “The Atomic Nucleus”, Equs.(8.5-7) quotes Feshbach and others as giving,

state	**a (fm)**	**V_0 (MeV)**
central	1.18	47
tensor	1.70	24

In addition, the triplet potential must depart from spherical symmetry. The angular dependencies consistent with a scalar potential are given by,

$$V_{strong} = -V_c(r) - \left[3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2\right] V_t(r) \qquad (2)$$

where $\sigma_{1,2}$ are the spin operators for the two nucleons. It may be shown that the expectation value of [...] is zero for the singlet spin state. Hence,

$$V_{strong}^{singlet} = -V_c(r) \qquad (3)$$

Conversely, for the triplet state with the spin oriented parallel to the polar z direction, the potential becomes,

$$V_{strong}^{triplet} = -V_c(r) - \left[3\cos^2(\theta) - 1\right] V_t(r) \qquad (4)$$

These potentials are claimed to be able to reproduce the properties of the deuteron, all low energy scattering data, and the binding energies of tritium, helium-3 and helium-4. In passing we note that the pion mass of ~137 MeV gives a decay length for the Yukawa potential of 1.44 fm, which equals the average of the two a-values for the central and tensor potentials. On the other hand, the mass of the rho meson (776 MeV) would imply a decay length of only 0.25 fm, which is not evident from the above fit to the data. However, we note that the above two Yukawa decay lengths may be written quite accurately as,

$$a_c = \left(\frac{1}{M_\pi} - \frac{1}{M_\rho}\right)\frac{\hbar}{c}, \quad a_t = \left(\frac{1}{M_\pi} + \frac{1}{M_\rho}\right)\frac{\hbar}{c} \qquad (1a)$$

To relate the above potentials to conventional measures of the strength of the nuclear force, we note that the pion exchange potential can be written in terms of the pion-nucleon vertex coupling constant as,

$$\text{exchange potential} = \alpha_s \frac{\hbar c e^{-m_\pi c r/\hbar}}{r}, \text{ where, } \quad \alpha_s = \frac{g_s^2}{4\pi} \cdot \left(\frac{M_\pi}{M_N}\right)^2 \qquad (5)$$

Thus, the strength of the singlet potential suggests,

(singlet) $$\alpha_s = V_{c0} a_c / \hbar c = 47x1.18/197.6 = 0.28 \qquad (6)$$

Alternatively, taking into account the different decay lengths (i.e. that a_c does not exactly correspond to the pion mass), and comparing strengths at, say, 1.44fm, gives,

(singlet) $$\alpha_s = (V_{c0}a_c / \hbar c)e^{1-1.44/a_c} = 47x1.18x0.8/197.6 = 0.225 \quad (7)$$

For the triplet state, the angular dependence of (4) means that it is not simple to identify an equivalent value for the strong coupling constant, g_s. Instead of (5) we would need to compare with the exchange potential of a meson of suitable spin, hence giving an angular dependence (hopefully) matching that of (4). A crude approximation will be given below.

Considering the deuteron, i.e. the triplet state, we immediately have a difficulty in deriving the energy levels corresponding to a potential given by (4) because of the angular dependence. The usual technique used to solve the Schrodinger equation, namely the separation of variables, fails in this case. Thus, the angular dependence of the eigenstates is no longer one of the Legendre polynomials. However, the Legendre polynomials, being a complete orthogonal set, provide a basis for expanding the angular dependence of the eigenstates (although the coefficients of each term in such an expansion will have different radial dependencies). We note that the angular dependence of the tensor potential is simply the second Legendre polynomial, P_2. This would correspond normally to L =2, i.e. an orbital D state. We conclude that the eigenstates of the deuteron (if indeed there are any) can be considered, roughly speaking, as a combination of an L = 0 (S) state and an L =2 (D) state. However, we will not follow this approach to demonstrate that the potential, (4), does indeed reproduce the binding energy of the deuteron (2.22 MeV). Instead we shall merely show that it is plausible that it would do so.

To this end we note that if a weighted average of the angular dependence of (4) is obtained by taking an expectation value with a D-state, i.e. we evaluate,

$$\frac{\frac{1}{2}\int_{-1}^{+1} P_2^3(c)dc}{\int_{-1}^{+1} P_2^2(c)dc} = \frac{4}{7} \quad (8)$$

then we might expect the energy levels to be approximated by those for a spherically symmetric potential given by,

$$V_{strong}^{triplet} = -V_c(r) - \eta V_t(r) \quad (9)$$

where $\eta = 4/7$. Solving the Schrodinger equation numerically for the spherically symmetric potential (9), and adjusting η until a binding energy of 2.22 MeV is reproduced, gives η = 0.477, which is not too far away from our estimated 4/7 = 0.57. Moreover, with this potential there are no other bound states. Hence it is plausible, as claimed by Evans / Feshbach et al that the true potential, (4), does reproduce the deuteron evidence correctly. *[Evans claims that only a ~3% admixture of D-wave is* *required to model the deuteron. This seems strangely small, far smaller than implied by the above estimate].*

[Aside: The implied effective value of the tensor coupling “structure constant” is $\alpha_S = \eta V_{t0} a_t / \hbar c \approx 0.1$. This compares with the singlet (=central) value of 0.225 to 0.28].

By scaling both the central and tensor potential down in proportion, we can determine when the deuteron just becomes unbound using these Yukawa potentials. We find this to be when they are decreased by 24% (as measured by V_0 or α_S) or about a 13% decrease in g_s.

Turing now to the dineutron and diproton, there is no need for such approximations since the potential is now the singlet potential, (3), and hence spherically symmetric. For the dineutron we can again attempt to solve the Schrodinger equation numerically using the potential (3). This confirms that there are no bound states. However, we may now explore the factor by which the potential (3) must be increased in order for a bound state to just exist. This turns out to be x1.1277, i.e. an increase in the strong force (as measured by V_0 or α_S) by 12.77% (or ~6% in g). This is very similar to the result obtained for the square well potential in Section 2.1.

For the diproton, the same technique can be used except that the (repulsive) Coulomb potential must also be included in the numerical solution. Clearly, therefore, there is no bound state. The factor by which the nuclear potential, (3), must be increased to yield a bound state in this case is x1.247, i.e. an increase in the strong force (as measured by V_0 or $\alpha_S = g^2 / 4\pi$) by 24.7% (or ~12% in g). This is rather larger than found for the square well potential, but is still quite modest in magnitude.

Davies [J.Phys.A.: Gen.Phys., Vol.5, August 1972, 1296-1304] has used a Yukawa potential together with a Bohr quantisation condition to estimate the changes to the strong coupling required for dinucleon stability. He finds considerably smaller changes induce deuteron instability or diproton/dineutron stability. These are included in the summary Table below. The reason for Davies’s different results is not clear but may be a consequence of the approximations of his method.

<u>Increases/Decreases in V or g^2 Resulting In Borderline Stability/Instability</u>

	Square Wells		**Two Yukawa Terms**	**Single Yukawa Term**
	Bradford	**Barrow & Tipler**	**Bradford**	**Davies**
Deuteron	-30%	-31%	-24%	-9%
Dineutron	+11%	+9%	+12.8%	+0.3%
Diproton	+16%	+13%	+24.7%	+3.4%

2.3 Justification Of Ignoring The Magnetic Dipole Interaction

The discussions of Sections 2.1 and 2.2 have ignored the magnetic dipole interactions between the nucleons. Both the proton and the neutron have a magnetic dipole, aligned respectively parallel and anti-parallel to the spin direction. Their magnitudes are respectively 2.793 and –1.913 times $e\hbar / 2M_p$. Thus, the magnetic dipoles of the nucleons are the order of a factor m_e / M_p smaller than the electron dipole moment. The potential energy due to two dipoles is,

$$\text{Energy} = \frac{\mu_0}{4\pi}\left[\frac{\overline{m}_1 \cdot \overline{m}_2 - 3(\overline{m}_1 \cdot \hat{r})(\overline{m}_2 \cdot \hat{r})}{r^3}\right] \tag{10}$$

Evaluating (10) at a spacing of, say, 1 fm for a neutron / proton pair with parallel spins gives an energy of –0.086 MeV (position vector normal to spins) or +0.172 MeV (position vector parallel to spins). Thus, the dipole energy is very small compared with the strong nuclear potentials at around a 1 fm spacing. Indeed, it is small compared with the Coulomb energy between two protons at 1 fm (1.44 MeV). This justifies neglecting the dipole interaction. However we note that at ~1/3 fm the dipole energy is of the same order as the deuteron binding energy (2.22 MeV). Nevertheless, including the dipole interaction potential explicitly in a numerical solution of the Schrodinger equation confirms that its effect on the bound state energies is small.

3. Implications Of An Unstable Deuteron Or A Stable Dineutron / Diproton

Authors of popular books on this issue have a tendency to deal with the effects of a stable diproton, or the absence of a stable deuteron, in a few throw-away remarks. The case of a weaker nuclear force, resulting in no stable deuteron, can indeed be dealt with swiftly (almost). The converse case, a stronger nuclear force resulting in a stable diproton, requires rather more careful consideration.

3.1 If The Nuclear Force Were Weaker ($g < 0.85g_{actual}$)

If the nuclear force were weaker ($g_s < 0.85g_{actual}$) the deuteron would not be stable. The effects of this are easy to understand. For the actual 'g_s', the deuteron becomes stable against photodisintegration at ~140 seconds after the Big Bang. This sets in motion the nuclear reactions which lead to the unstable neutrons finding sanctuary in stable nuclei, predominantly helium-4, with trace amounts of deuterium, helium-3 and lithium-7 (see Chapter 6). This sequence of reactions could not initiate without stable deuterons forming the first step. Thus, for our reduced strength nuclear force, no nuclei would be created in the immediate aftermath of the Big Bang itself. The neutrons would be doomed to decay, their half-life of ~15 minutes being unaffected since this is controlled by the weak nuclear force. Thus, after a few hours, the only baryons left would be free protons.

Subsequent conditions in the universe would still be conducive to star formation – at least the first generation of stars. However, the formation of deuterons is also the first reaction in the nuclear burning of stars. The absence of a stable deuteron would therefore prevent the star producing any heat – at least after its gravitational heating had been exhausted. More importantly, the absence of a stable deuteron would mean that no other elements could be formed. The universe would be doomed to consist entirely of hydrogen forever[2]. Clearly, this is not a promising recipe for an interesting universe.

3.2 If The Nuclear Force Were Stronger ($g > 1.1g_{actual}$)
If the nuclear force were stronger ($g > 1.1g_{actual}$) the diproton and dineutron would be stable. The assessment of this condition in most popular books on this subject runs as follows:-

(a) Barrow & Tipler (P.322) and Rees (P.55) argue that the stability of the diproton would lead to all the protons being thus combined in the early universe (i.e. in the first few minutes). The same argument is presented in Carr & Rees (Nature, **278**, 12 April 1979), which states, "If g_s were even merely a few per cent larger there would be 100% cosmological helium production because He^2 would be bound and deuterium could form by $p + p \to He^2 + \gamma$; and $He^2 \to D + e + \overline{\nu}_e$ even if there were no frozen-out neutrons". This would mean that no hydrogen would exist in the universe. Hence, stars would not be fuelled by hydrogen. Moreover, no hydrogen compounds (e.g. water, hydrocarbons, etc) could ever exist. Thus, carbon-based life-forms of the type we know would be impossible.

(b) If any protons survived the early universe, Davies (P.71) argues that the rapid formation of diprotons, followed by their rapid inverse beta decay into deuterons, would provide a very fast initial step in the nuclear burning sequence. Specifically, he suggests it would be about 10^{18} times faster than the conventional stellar formation of deuteron via the weak interaction. Thus, he concludes that a stable diproton would lead to "catastrophic hydrogen consumption and energy release". Precisely what this phrase means is not clear, e.g. whether Davies envisages all hydrogen being thus consumed – and hence the same implications for subsequent chemistry as in (a), above.

Both these scenarios are mistaken. Diprotons would not actually form during the BBN period because its rate of formation would be too slow. The existence of a stable diproton would affect stars very significantly. Nevertheless stable, long-lived stars with a surface temperature consistent with that required to nurture life would still be possible. This will be explained in detail in Chapter 9C.

Before closing this Chapter we consider what effect a change in the strength of the nuclear force has on the binding energies of nuclei. We have already made use of this in Chapter 9.

4. Effects Of A Stronger Nuclear Force On The Binding Energies Of Nuclei
As the nuclear force increases in strength it is reasonable to expect the binding energy of the ground state of any given nucleus to increase also. [It is important to specify the ground state when making this statement. As the strength of the nuclear force is increased, new excited states may arise which will have arbitrarily small binding energies for suitably chosen values of g_s]. The question is, exactly how does the binding energy increase with g_s? In the case of the heavier nuclei, Davies (J.Phys.A, Vol.5, Aug.1972, 1296-1304) argues by analogy with the hydrogen atom that the

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[2] To be more complete, the universe would consist of atomic hydrogen, ionised hydrogen, i.e. protons and electrons, plus photons and the three types of neutrino.

binding energy will be proportional to g_s^4. The precedent is the nuclear shell model which assumes that the nucleons move as if in a central field which results from the combined effects of all the nucleons. Thus, if the effective potential varied as 1/r, the hydrogen atom solution would indeed be a good guide. The energy levels of the hydrogen atom are $|E_n| = \alpha^2 m_e c^2 / 2$ where $\alpha = e^2 / \hbar c$, and hence $|E| \propto e^4$. Since the Coulomb potential is e^2 / r and the strong force potential (in the approximation assumed) would be $g_s^2 \hbar c / r$, it follows that the nuclear binding energies would be proportional to g_s^4, as claimed.

However, this argument does not seem convincing to me. The shell model has been used successfully to account for the order of the energy levels of a nucleus. However, it is predominantly the angular moment states (orbital and spin) which give rise to this order (1s, 1p, 1d, 2s, 1f, 2p, 1g,…). The same order is found for deep square well potentials and oscillator type potentials (see Evans P.361). Thus, the success of the shell model in no way motivates the assumption of a 1/r potential. In fact, given that the range of the strong force is only 1-2 fm, the influence of the exponential decay in Yukawa type potentials cannot be ignored for large nuclei. This is important since an energy spectrum like that of the hydrogen atom is really a consequence of the long range of the Coulomb force. One consequence of this is, for example, that no matter how small the constant, α, in a potential α/r, there are still an infinite number of bound states (with binding energy proportional to $1/n^2$, with n =1, 2, 3,…). In contrast, for a Yukawa potential, $\alpha e^{-mr} / r$, there will be no bound state if α is sufficiently small, and just one bound state for slightly larger α, followed by a range of α for which there are just two bound states, etc. In this respect the Yukawa potential behaves like a square well potential, revealing that the feature which gives rise to energy spectra like these is the short-range nature of the potential rather than its detailed shape. Irrespective of this argument for large nuclei, it is reasonably well established for small nuclei that square well or Yukawa potentials provide a good guide to the energy levels.

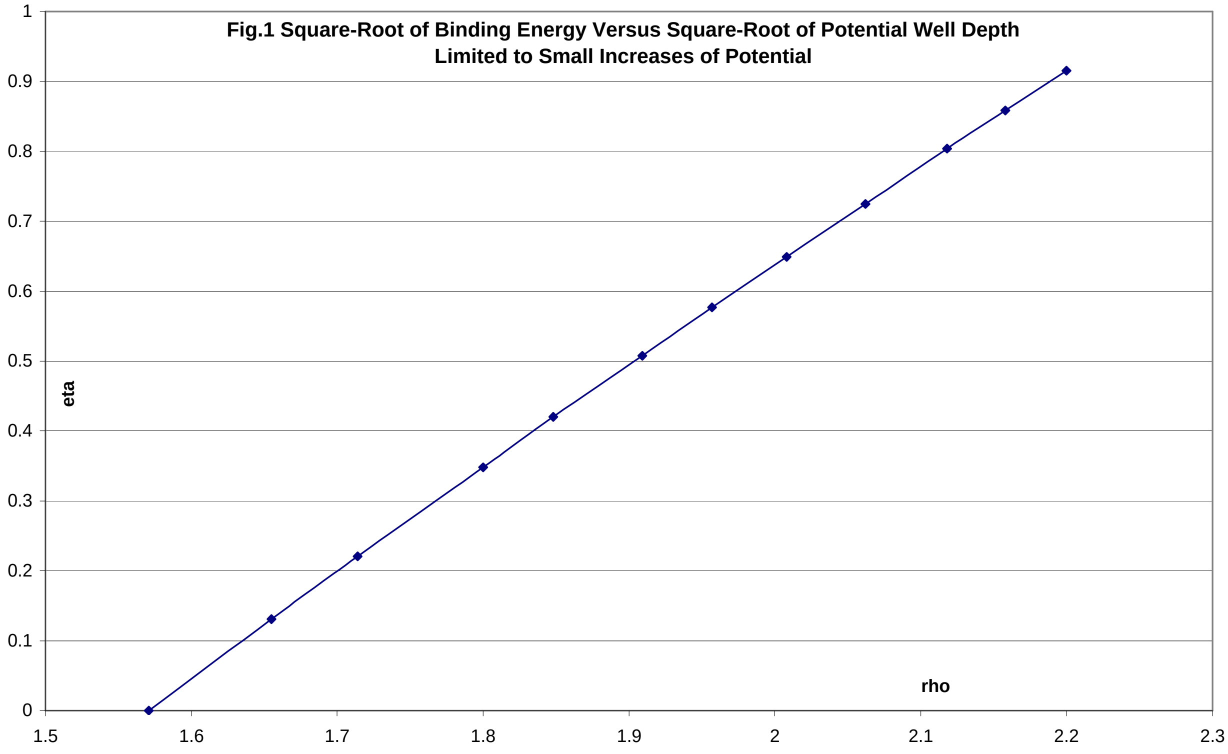
Hence, in contrast to Davies (J.Phys.A, Vol.5, Aug.1972, 1296-1304) we shall assume that the 3D Schrodinger equation with a square-well potential gives the better guide to the dependence of the binding energy on coupling constant. The well depth is taken to be proportional to the square of the coupling constant, i.e. $V \propto \alpha_s \propto g_s^2$ as per the Yukawa potential. As derived in Schiff, the energy levels of the 3D square well potential are given by,

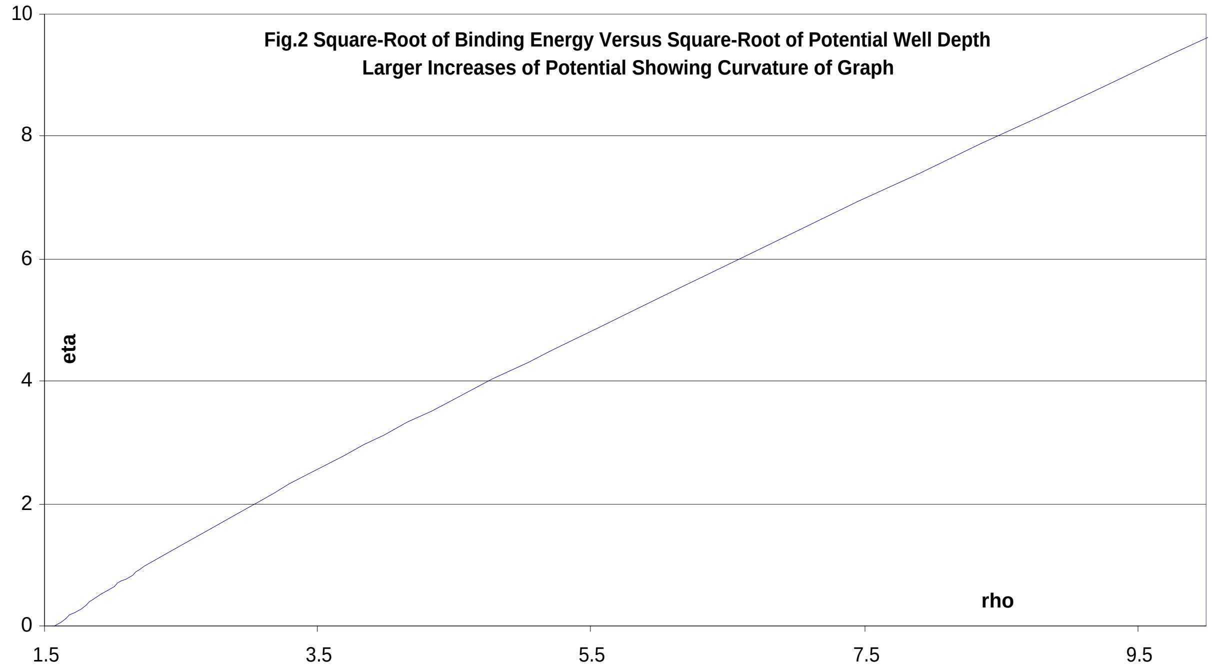
$$\xi \cot \xi = -\eta \quad \text{with} \quad \xi^2 + \eta^2 = \rho^2 \quad \text{where} \quad \rho^2 = 2ma^2 V / \hbar^2 \tag{11}$$

$$\text{where, } \xi = a\sqrt{2m(V - |E|)} / \hbar \quad \text{and} \quad \eta = a\sqrt{2m|E|} / \hbar \quad \text{and} \quad m = M_p / 2 \tag{12}$$

Thus, the discrete solutions to the simultaneous equations (11) give the energy levels via (12). There are no solutions for $\rho < \pi / 2$ and just one solution for $\pi / 2 < \rho < 3\pi / 2$ and exactly two solutions for $3\pi / 2 < \rho < 5\pi / 2$, etc. Since we are considering relatively small variations of the coupling constant, from a value which has a single bound state of the pn system, we shall confine attention to the range $\pi / 2 < \rho < 3\pi / 2$.

The following graphs show how the bound state value for the dimensionless parameter η ($\propto \sqrt{|E|}$) varies with the dimensionless parameter ρ ($\propto \sqrt{V} \propto g_s$):-





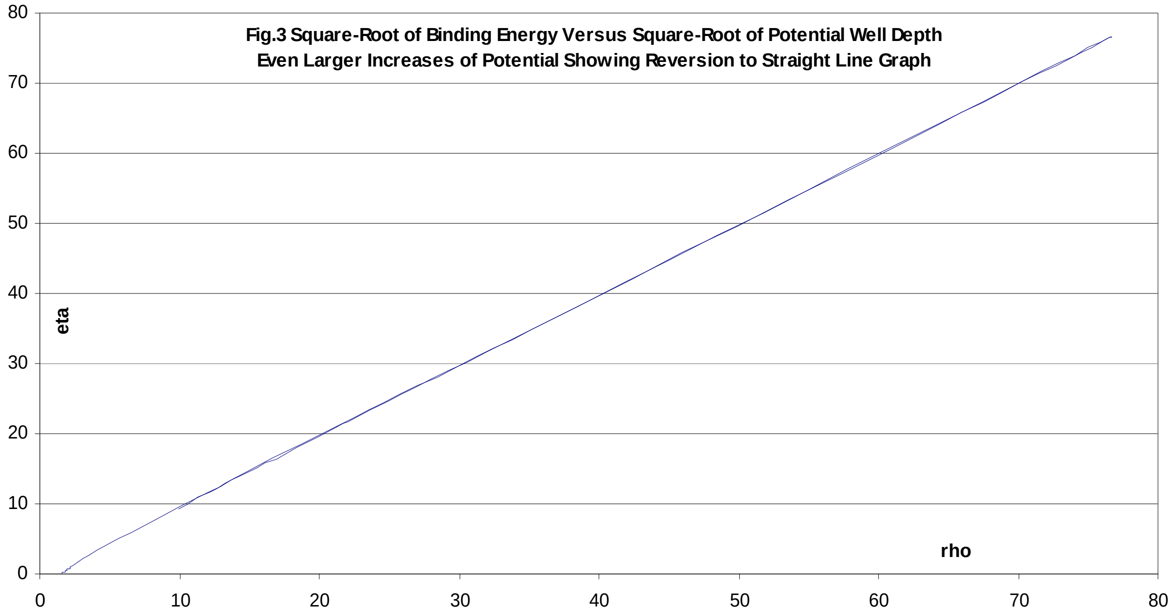


Figure 1 shows that for small increases in potential well depth, the unique bound state solution has $\eta \approx 1.46(\rho - \pi/2)$. For larger increases in well depth, straddling the occurrence of a second bound state (i.e. taking ρ above 3π/2), the curvature in the graph of the ground state η versus ρ becomes discernible. However, for still deeper potential wells, the ground state approaches the solution $\eta \approx \rho$. The case of interest is the first (modest increases in nuclear strength). Thus we have,

$$\sqrt{|E|} \propto g_s - g_s^{MIN} \qquad \text{or} \qquad |E| \propto \left(g_s - g_s^{MIN}\right)^2 \tag{13}$$

where g_s^{MIN} is the smallest coupling constant which results in a bound state (i.e. corresponding to a value of V which gives ρ = π/2). In the case the deuteron, the weakest nuclear force which just binds the deuteron has g_s reduced by ~15%, hence,

$$g_s^{MIN} = 0.85 g_s^{actual} \tag{14}$$

We estimate that a 10% increase in g_s is just sufficient to bind the diproton (the average of the results we obtained above using the square well potential and the Yukawa potential). Hence in a universe with $g_s = 1.1 g_s^{actual}$ the binding energy of the deuteron is estimated to increase by a factor,

$$\frac{|E|}{|E_{actual}|} = \frac{(1.1 - 0.85)^2}{(1.0 - 0.85)^2} = 2.78 \tag{15}$$

This contrasts sharply with Davies's assumption that $|E| \propto g_s^4$, which would increase the binding energies by a factor of only $1.1^4 = 1.46$.

The deuteron binding energy in our alternative universe is simply the scaling factor of Equ.(15) times its actual binding energy (2.22 MeV).

The nn binding energy in our alternative universe is found using Equ.(12) together with $\eta \approx 1.46(\rho - \pi/2)$ and the singlet potential a = 2.6 fm and V = 13.68MeV (a^2V = 92.5 MeV.fm), as per the Table in Section 2, with the potential V being scaled by $\left(g_s / g_s^{actual}\right)^2$. The first pp binding energy is the nn binding energy less 0.55 MeV for the Coulomb effect. Note that this results in a larger coupling constant being required before the diproton is bound (i.e. an increase in g_s of about 20% rather than 10%). This is simply because of the different means of estimating the effect of the Coulomb potential. The alternative method, i.e. reducing the strong coupling constant by ~2%, gives a less marked effect on the binding energies (see the figures in brackets). We have to accept that these are, after all, just rough estimates.

The following Table summarises the binding energies for various increases in g_s:-

g_s / g_s^{actual}	$\lvert E\rvert / \lvert E_{actual}\rvert$ deuteron Equ.(15)	cf. $\left(g_s / g_s^{actual}\right)^4$	Binding energy, MeV		
			Deuteron (triplet state)#	Dineutron	Diproton
1.0	1.0	1.0	2.22	*unbound*	*unbound*
1.054	1.85	1.23	4.1	0	*unbound*
1.1	2.78	1.46	6.2	0.06	~0
1.15	4.00	1.75	8.9	0.27	~0 (~0.15?)
1.2	5.44	2.07	12.1	0.62	>0.07 (~0.45?)
1.25	7.11	2.44	15.8	1.12	>0.57 (~0.87?)
1.3	9.00	2.86	20.0	1.77	>1.22 (~1.45?)
1.4	13.44	3.84	29.8	3.50	>2.95 (~3.1?)

**This applies only to nuclei which are stable in this universe in any case (i.e. E_{actual} < 0). For nn and pp see below.*

#The singlet state will also be bound whenever the diproton is bound, and with a binding energy between that of the dineutron and the diproton.

In the above scaling we have ignored complications such as the components of the nuclear force with different angular dependencies (central and tensor forces). Implicitly we are assuming that both of these components scale equally. To check our scaling formula for the binding energy, Equ.(13), we have run our numerical solution of the Schrodinger equation for the two Yukawa term model, in the spherically symmetric approximation (see Section 2.2). Both terms have been scaled by the same factor. The comparison between the numerical binding energy for the deuteron (triplet state) and the result of using Equ.(13) (with $g_s^{MIN} = 0.8718$, which is the limit for deuteron binding with this potential) is:-

Factor on V or α or g_s^2	**Factor on g_s**	**Numerical Solution Binding Energy, MeV**	**Equ.(13) Binding Energy, MeV**
0.76	*0.8718*	*0 (just unbound)*	*0*[(1)]
0.8	0.8944	0.18	0.07
0.9	0.9487	0.92	0.80
1	*1*	*2.22*	*2.22*[(1)]
1.1	1.0488	4.10	4.23
1.2	1.0954	6.59	6.75
1.3	1.1402	9.69	9.73
1.69	1.3	27.8	24.8

[(1)]*These values are constrained.*

This confirms that Equ.(13) is a reasonably accurate scaling formula for the binding energy (at least for the modest range of variation in g_s of interest).

We note that a formula like Equ.(15) can also be applied to estimate the binding energies of the heavier nuclei in out alternative universe, in the form,

$$\frac{|E|}{|E_{actual}|} = \frac{\left(g_s - g_s^{MIN}\right)^2}{\left(g_s^{actual} - g_s^{MIN}\right)^2} \tag{16}$$

However, the heavier nuclei are more tightly bound than the deuteron, so we may expect a smaller value for g_s^{MIN}. We have seen how this can be estimated already in Chapter 9 (and have found that generally $g_s^{MIN} \approx 0.6 g_s^{actual}$).

5. Implications of a Bound Diproton?

This will be considered in Chapter 9C. However, by way of a taster, we have estimated the earliest time at which the temperature has dropped sufficiently to render diprotons stable to photodisintegration. This uses the same method as in the Tutorial Chapter 6. The diproton becomes more stable the greater is its binding energy, i.e. the greater is g_s. Thus,

<u>Binding Energies Against Notional Increase In Strong Coupling Constant (g_s): Times At Which Dinucleons Are Stable Against Photodisintegration</u>

g_s / g_s^{actual}	**Binding energy, MeV**			**Time for Stability Against Photodisintegration**	
	Deuteron (triplet)	**Dineutron**	**Diproton**	**Deuteron (triplet) <u>sec</u>**	**Diproton <u>mins</u>**
1.15	8.9	0.27	0.15	8.3	460
1.2	12.1	0.62	0.45	4.5	50
1.25	15.8	1.12	0.87	2.6	14
1.3	20.0	1.77	1.45	1.6	5
1.4	29.8	3.50	3.1	0.7	1.1

In this universe the deuteron become stable at around 3 minutes, so even a 15% increase in g_s dramatically shortens this time. For a 30%-40% increase in g_s the diproton becomes stable at around the same time as the deuteron does in this universe (i.e. a few minutes). Consequently, if the pp capture rate were comparable with the np capture rate, all the protons left over after deuteron production would be cooked into diprotons. So, ***if*** the pp capture rate were comparable with the np capture rate the scenario described by many previous authors, as outlined in Section 3.2, would indeed be realised. But it isn't, as we shall see in the next Chapter (9C).

4. Conclusions

We have seen in Section 2 that decreasing the strength of the strong nuclear coupling constant (g_s) by about 15% would result in the deuteron being unbound. Conversely, an increase in g_s of ~10% would be sufficient to cause the diproton to be stable, and an increase in g_s of only ~5% is sufficient to stabilise the dineutron.

If g_s were reduced by more than ~15%, and hence the deuteron did not exist, the implications for the universe would be catastrophic. The lack of deuterium would remove the pathway for the formation of heavier elements. Thus, whilst heavier nuclei might still be stable, there would be no means of producing them.

The implications of increasing g_s by more than 10%, resulting in a bound diproton, are discussed in Chapter 9C. Anticipating the conclusions of Chapter 9C, we shall find that the existence of the diproton is probably not catastrophic for the evolution of a complex, chemically rich, universe.

Hence, we conclude that, as regards di-nucleon stability, g_s exhibits a single-sided fine tuning of Type C1, namely $g_s > 0.85 g_s^{actual}$. The degree of fine tuning in g_s is comparable to that found in Chapter 9 regarding the stability of the heavier elements (carbon to calcium).

We have taken the opportunity in this Chapter to derive an expression for how the binding energy of a nucleus might scale as g_s is varied. Our formula is,

$$\frac{|E|}{|E_{actual}|} = \frac{\left(g_s - g_s^{MIN}\right)^2}{\left(g_s^{actual} - g_s^{MIN}\right)^2} \qquad (16)$$

This is believed to be appropriate for nuclear forces which approximate to finite range potentials, e.g. square-well potentials or Yukawa potentials. The characteristic of finite range potentials is that they have no bound states if the magnitude of the potential is below some lower bound. This lower bound corresponds to g_s^{MIN}, the smallest coupling strength which results in the nucleus in question being bound. For deuterium $g_s^{MIN} \approx 0.85 g_s^{actual}$, whereas for heavier elements it is roughly $g_s^{MIN} \approx 0.6 g_s^{actual}$. Equ.(16) is in contrast to the scaling which would apply for an inverse square force law, i.e. for $V \propto g_s / r$. Such an infinite range potential has bound states however small is g_s. The binding energy for this potential is proportional to g_s^4. Equ.(16) is generally more sensitive to changes in g_s than is g_s^4.