Chapter 9CS: The Fine Tuned Strong Force? - The Effect of Diproton Stability on the Universe

Abstract: *By calculation of the proton-proton capture cross section it is shown that postulated increases in the strong nuclear force (g_S) would not lead to significant production of diprotons in the immediate post-Big Bang period. The stability of diprotons would lead to a reaction pathway for converting protons to deuterons ~10^{10} times faster than the usual weak capture reaction in typical stellar interiors. This would prevent stars of the familiar hot, dense type from occurring in the universe. However, stars with temperatures and densities sufficiently reduced so as to offset the faster reaction pathway to deuterium appear to meet elementary stability criteria. An anthropic upper bound to the strong force to ensure diproton instability appears to be unfounded.*

1. INTRODUCTION

Many of the basic parameters of physics cannot be varied greatly from their actual values without fatally compromising the production of a universe within which highly complex structures, including life, can evolve [e.g., Ref.1]. One example is the strong nuclear coupling constant, g_s. A reduction in g_s of ~15% would result in the deuteron being unbound. Since the initial step of stellar nuclear fusion is $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{D} + \mathrm{e}^+ + \nu_e$, stars would be robbed of their power source and be still-born. Moreover, the only chemical element in the universe would be hydrogen, making all chemistry, and hence chemistry based life, impossible. Conversely, an increase in g_s of ~8% would render both the dineutron and the diproton (${}^2_2\mathrm{He}$) stable. The consequences of this have been claimed to be that, "all the hydrogen in the universe would have been burnt to ${}^2_2\mathrm{He}$ during the early stages of the Big Bang and no hydrogen compounds or long-lived stable stars would exist today" [Ref.1]. This claim is commonly asserted, for example in [Ref.2], "*Diproton formation* would have happened readily in the early universe, so that no hydrogen would remain to provide the fuel in ordinary stars, and water could never have existed." Similarly, in [Ref.3], "If g_s were even merely a few per cent larger, there would be 100% cosmological helium production because He^2 would be bound, and deuterium could form by $p + p \rightarrow He^2 + \gamma$, followed by $He^2 \rightarrow D + e^+ + \nu_e$".

However, this scenario would be realised only if the proton-proton capture reaction were sufficiently fast to ensure that virtually all the free protons were captured before the diminishing temperature and density in the minutes following the Big Bang led to the reaction's termination. The "diproton disaster" described above appears to have been based on a false analogy with neutron-proton capture to form deuterons. In this paper we shall show that, in contrast, the proton-proton capture reaction rate is too slow to lead to any significant production of diprotons. The universe therefore remains unaffected by diproton stability until local gravitational clumping initiates star formation.

2. THE PROTON-PROTON CAPTURE CROSS-SECTION

In what follows we assume g_s to have been increased sufficiently for the diproton to be bound. For definiteness we shall consider increases of 20%, 30% and 40%. The proton-proton capture cross section (σ_{pp}^{cap}) is smaller than that for neutron-proton (σ_{np}^{cap}) capture for three reasons:-

1) Most obviously, the Coulomb barrier reduces σ_{pp}^{cap}. This is a small effect at Big Bang nucleosynthesis temperatures ($\sim 10^9$K), but accounts for several orders of magnitude reduction in the corresponding reaction rate at, say, central solar temperatures ($\sim 14 \times 10^6$K).

2) At the non-relativistic energies of interest (≤ 1 MeV), the neutron-proton capture cross section can be estimated simply from Schrodinger matrix elements [see for example, Refs.4, 5]. The dominant contribution arises from the magnetic dipole term, i.e. the coupling between the nuclear spins and the magnetic component of the electromagnetic field. However, this matrix element is proportional to the difference between the magnetic dipole moments of the two particles, and is therefore zero for identical particles, e.g. for pp capture.

3) The second order term contributing to neutron-proton capture is the electric dipole interaction, i.e. the coupling between the charge and the electric component of the electromagnetic field. (The electric dipole cross-section is about an order of magnitude smaller than the dominant magnetic dipole cross-section at $\sim 10^9$K, and about three orders of magnitude smaller at $\sim 10^7$K). Because the deuteron is a spin triplet (^{3}S), and because the electric dipole interaction Hamiltonian (H_I^D) is proportional to $r\cos\theta$ and does not affect the spin, the only non-zero matrix element is for an initial spin-triplet P-wave state, i.e. $< {}^3S(\text{bound}) | H_I^D | {}^3P(\text{free}) >$. In contrast, the diproton is a spin singlet state (^{1}S), as required by the exclusion principle. The relative weakness of the singlet nuclear force is the reason why, in this universe, the diproton does not exist. The electric dipole matrix element would be $< {}^1S(\text{bound}) | H_I^D | {}^1P(\text{free}) >$, but of course the singlet P-state cannot exist for identical fermions. Hence, the second order term contributing to σ_{np}^{cap} is also zero for σ_{pp}^{cap}.

We conclude that the lowest order non-zero term contributing to σ_{pp}^{cap} must be the electric quadrupole term, i.e. $< {}^1S(\text{bound}) | H_I^Q | {}^1D(\text{free}) >$, where, $H_I^Q \propto r^2 P_2(\cos\theta)$. Thus, it is clear that $\sigma_{pp}^{cap} << \sigma_{np}^{cap}$.

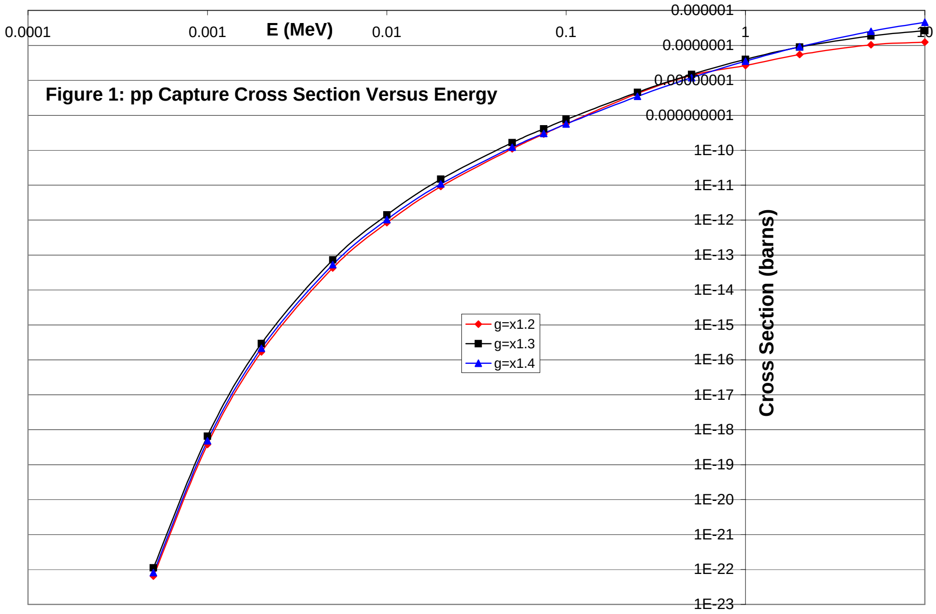
Standard methods [Refs.4, 5] may be used to derive an analytic expression for the cross section if the Coulomb interaction is ignored, and in the zero-range approximation,

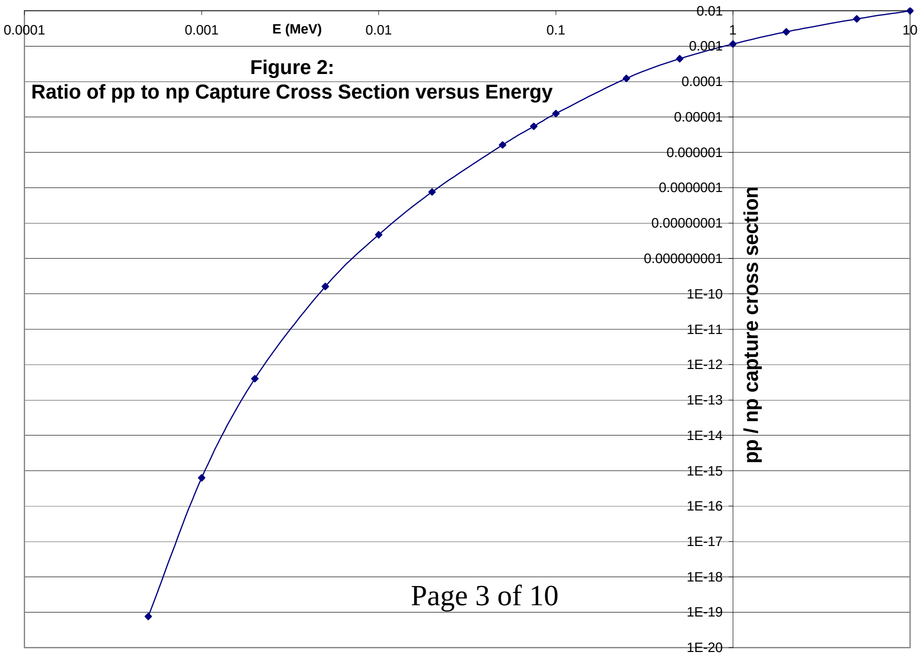
$$\sigma_{pp}^{cap} = \frac{64\pi\alpha(\hbar c)^2 B^{1/2} E^{3/2}}{15(M_p c^2)^3 (E+B)} \qquad \text{(no Coulomb barrier)} \qquad (1)$$

where B is the postulated diproton binding energy and E is the sum of the two incident protons' kinetic energies in the centre-of-mass system. When the Coulomb barrier is included, the cross section diminishes at sufficiently low energies proportionally to,

$$\sigma_{pp}^{cap} \propto \exp\left\{-\pi\alpha\sqrt{M_p c^2 / E}\right\} \quad \text{(E ~ 1 keV or smaller)} \qquad (2)$$

in the usual way. We have chosen to evaluate numerically the Schrodinger wavefunctions including the Coulomb potential, using a nominal singlet nuclear potential 'square well' with a = 2.4 fm and V_0 = 16.1 MeV (Refs.4, 5 and noting that great accuracy is not our objective). The potential well depth is increased by factors of 1.2^2, 1.3^2 or 1.4^2 to derive the cross section for diproton formation. [These correspond to diproton binding energies of 0.6, 2 and 4 MeV respectively]. The numerical results have the low energy behaviour given by (2) and reproduce (1) when the Coulomb interaction is removed. The cross section (in barns) is plotted against E in Figure 1, and as a fraction of the neutron-proton capture cross section in Figure 2.





At ~0.1MeV, the proton-proton capture cross section is about 5 orders of magnitude smaller than the neutron-proton capture cross section. At ~1keV the difference is about 15 orders of magnitude. The latter is due largely to the Coulomb barrier. However, the small proton-proton capture cross-section compared with that of neutron-proton capture at ~0.1MeV is mostly due to the fact that the former involves identical particles.

A reasonably good closed-form approximation to the numerical cross-section results is,

$$\sigma_{pp}^{cap} = \frac{64\pi\alpha(\hbar c)^2 B^{3/2} E^{1/2}}{15(M_p c^2)^3 (E+B)} \exp\left\{-\pi\alpha\sqrt{M_p c^2 / E}\right\} \tag{3}$$

Note that (3) differs from the simple product of (1) and (2) by an additional factor of B/E, motivated simply to improve agreement with the numerical results shown in Figure 1.

The overall reaction rate for a thermal distribution of proton energies is found by integrating the monochromatic rate, determined from (3), appropriately weighted by the Maxwell distribution. Using a first order approximation for the resulting 'Gamow peak' integral yields a rate at temperature T given by,

$$R[T] \approx \frac{16\cdot 2^{2/3}}{15}\pi b^{7/3}\alpha A\left(\frac{\hbar}{M_p c}\right)^2\left(\frac{c}{M_p c^2}\right)\cdot\sqrt{\frac{B}{M_p c^2}}\cdot\frac{\exp\{-f_{min}\}}{(kT)^{1/6}} \tag{4}$$

where, $f_{\min} = 3\left(\dfrac{b}{2\sqrt{kT}}\right)^{2/3}$ where, $b = \pi\alpha\cdot\sqrt{M_p c^2}$ (5)

and where E << B is assumed and A is Avogadro's number (~6 x 10^{29} /m^3). Equ.(4) gives the reaction rate in $s^{-1}(\text{mole/cm}^3)^{-1}$ for kT in MeV. Using B = 2 MeV for illustration (i.e. for a 30% increase in g_s), we deduce an approximate reaction rate at temperature T,

$$R[T] = 6.7\frac{e^{-f_{min}}}{[kT(\text{MeV})]^{1/6}}\ s^{-1}(\text{mole/cm}^3)^{-1} \tag{6}$$

3. ARE DIPROTONS FORMED AFTER THE BIG BANG?

It is convenient to express results in terms of time (t), taken as correlated with temperature according to $T(K) = 10^{10}/\sqrt{t(sec)}$. The baryon-photon ratio is taken to be 2 x 10^9. The diproton reaction times over the first hour following the Big Bang, derived from Equ.(6), are given in Table 1.

Table 1: Diproton Formation Reaction Times

t (sec)	kT (MeV)	Reaction Rate $s^{-1}(mole/cm^3)$	proton density $/m^3$	Reaction rate /s	Reaction Time, s	Reaction time / t
1	9.E-01	1.E+00	8.E+27	2.E-02	5.E+01	5.E+01
10	3.E-01	8.E-01	3.E+26	4.E-04	3.E+03	3.E+02
30	2.E-01	6.E-01	5.E+25	5.E-05	2.E+04	7.E+02
50	1.E-01	5.E-01	2.E+25	2.E-05	6.E+04	1.E+03
100	9.E-02	3.E-01	8.E+24	5.E-06	2.E+05	2.E+03
200	6.E-02	2.E-01	3.E+24	1.E-06	9.E+05	4.E+03
300	5.E-02	2.E-01	2.E+24	5.E-07	2.E+06	7.E+03
500	4.E-02	1.E-01	7.E+23	2.E-07	6.E+06	1.E+04
1000	3.E-02	9.E-02	3.E+23	4.E-08	3.E+07	3.E+04
2000	2.E-02	5.E-02	9.E+22	7.E-09	1.E+08	7.E+04
3000	2.E-02	3.E-02	5.E+22	3.E-09	4.E+08	1.E+05
5000	1.E-02	2.E-02	2.E+22	8.E-10	1.E+09	3.E+05

The condition for reaction freeze-out by cosmic expansion is that the reaction time exceed 1/H ~ 2t, i.e. that the last column in Table 1 should exceed ~2. Hence we see that the diproton reaction is frozen out at all times after ~1 second, and indeed somewhat before that. The situation contrasts with that for neutron-proton capture. Consistent with actual Big Bang nucleosynthesis, Figure 2 implies that the reaction times for the latter are shorter than t during this period.

To complete the argument that diprotons would not be a product of Big Bang nucleosynthesis we now demonstrate that diprotons would photodisintegrate prior to 1 second. All neutrons will be assumed to have combined as deuterons before the time at which diprotons become stable against photodisintegration. (The increased strength of the nuclear force will increase the binding energy of the deuteron, which will thus always be stable at higher temperatures than the diproton). The maximum possible diproton to photon ratio is thus $0.75 / (2 \times 1.9 \times 10^9) = 2 \times 10^{-10}$, noting that the proportion of remnant protons (~75%) is not affected by the change in g_s.

The simplest estimate of the temperature at which diprotons will be stable against photodisintegration is obtained by equating the maximum possible diproton:photon ratio to the fraction of photons with energies sufficient to cause photodisintegration, i.e. greater than B. This fraction is, from the blackbody photon density,

$$0.416\int_{x_1}^{\infty}\frac{x^2 dx}{e^x - 1} \approx 0.416\left[2 + 2x_1 + x_1^2\right]e^{-x_1} \qquad \text{where, } x_1 = B/kT >> 1$$

Thus we find $x_1 = 28.22$ and the earliest times at which diprotons are stable are,

g / g_{actual}	1.2	1.3	1.4
B (MeV)	0.6	2	4
T (K)	2.4×10^8	8.2×10^8	1.65×10^9
t (sec)	1,700	150	37

Thus, diprotons do not become stable until well after ~1 second. The proton-proton capture reaction is therefore frozen out before diprotons become stable. There would therefore be no large scale production of diprotons in the immediate post-Big Bang period.

4. IS g_s FINE TUNED AGAINST INCREASES?

It would appear that the stability of the diproton makes no difference to the primordial universe, which would retain its approximately 75%:25% ${}^1_1\mathrm{H} : {}^4_2\mathrm{He}$ constitution. However, the effect on star formation would obviously be dramatic. For example, at a solar central temperature of (say) 15 x 10^6 K, the pp capture reaction rate would be about 2 x 10^{-5} $s^{-1}(\mathrm{mole/cm^3})^{-1}$, and a solar central proton density of 4 x 10^{31} m^{-3} would lead to a pp reaction time of less than an hour. Thus, the rate of production of deuterons would be controlled by the rate of the weak decay of the diproton. Even if this were the order of a year, the overall deuteron production rate would be ~10^{10} times faster than the usual weak capture reaction $p + p \to D + e^+ + \nu_e$ under the same conditions.

However, the diproton reaction is so rapid under solar temperature and density conditions that it would be explosive. Stars of this type could not form. It is clear, therefore, that the universe would be radically different from the actual universe once incipient star formation started. What is not clear is whether stable, long lived, stars would form with temperatures and densities suitably reduced so as to offset the faster reaction pathway to deuterium. We do not pretend to provide a definitive answer to this question here. However, the answer appears to be less clear cut than is often suggested. To see this, consider an hypothesised star with a central temperature of 10^6 K.

There are several elementary constraints which a stable star must respect. For example, dynamical stability requires that the radiation pressure within the star should not be too much larger than the gas pressure. This constraint leads to the familiar upper bound on stellar masses, of roughly 50 solar masses, a limit which will also apply in our alternative universe. This constraint can also be written as a lower bound on the gas density required for stability, namely $\rho > 0.1 M_p \left(\frac{kT}{\hbar c} \right)^3$, which is 0.015 kg/m^3 for our example.

The star must also be able to transport heat efficiently enough to balance the rate of nuclear heat generated. At the centre of the star there is a maximum power density consistent with purely radiative heat transfer, i.e., $\varepsilon_v < \varepsilon_v^{max} = 4\pi c G \rho / \kappa$, where κ is the opacity (the subscript $_v$ denotes power per unit volume, rather than per unit mass). Since the power density depends upon the square of the proton density, this limit on power density results in an upper limit on proton density. It evaluates to about 2.6 kg/m^3 for our example. This based on the diproton reaction rate from Equ.(6) together with the reaction sequence given in the Appendix. Of significance is the fact that at 10^6 K, and a density of 2.6 kg/m^3, the opacity of pure hydrogen is only beginning to rise above the lower bound provided by Thompson scattering (namely ~0.15 m^2/kg, compared with the Thompson opacity of 0.034 m^2/kg).

Satisfying hydrostatic equilibrium and heat balance everywhere within the star would determine the unique central density for a given central temperature (if any stable solution exists). In the absence of a complete stellar model, however, we have derived the possible range within which the central density must lie, namely between 0.015 kg/m^3 and 2.6 kg/m^3.

An estimate for the star's lifetime is the reaction time based on Equ.(5). Using the above limiting densities suggests lives between 200 Myrs and 20 Byrs. The mass of our star may be estimated from $kT_c \approx 0.24 G M_p M^{2/3} \rho_c^{1/3}$, which is an approximate expression of being gravitationally bound (the Virial Theorem), except that average quantities have been replaced by their central values. This implies masses of between ~3 and ~50 solar masses (the latter being by construction of the lower bound density).

Estimation of the surface temperature is more contentious. To do so we have assumed that one quarter of the star's mass is involved in nuclear reactions at the central rate. This results in luminosities of 300 to 8,000 times solar luminosity. The radius is estimated using $R \approx 2.5\left(\frac{M}{\rho_c}\right)^{1/3}$, which suggests sizes 50 to 650 times solar size. Finally, the preceding results imply a surface temperature between 1,400 K and 7,600 K.

Of course we have not definitively established that such stars could exist. To do so would require explicit demonstration that hydrostatic equilibrium and heat balance were respected at all points in the star, i.e. a complete stellar model. Fortunately this is not where the burden of proof lies. We have seen that the elementary stability constraints can be consistent with a star which is sufficiently long lived, sufficiently luminous, and has a suitable range of surface temperatures, to mimic the actual conditions of the true universe to some approximation. In view of this, the burden of proof lies with any contention that diproton stability is anthropically catastrophic.

It is rather remarkable that a reaction which is many orders of magnitude faster than the usual weak pp capture reaction can be tamed and result in a stable star simply by reducing the temperature and density. It is reasonable to ask whether this trick could be repeated for an even faster reaction. The limit to this may be that at still lower temperatures the opacity will start to climb steeply (Kramer's opacity $\propto 1/T^{7/2}$). This will severely restrict the power density which can be balanced by purely radiative heat transfer. It appears that our example is close to this limit.

It is amusing to speculate how physicists in an alternative universe, containing stars like that of our example, might view their situation. They might point to three remarkable 'coincidences'. The first would be that the Thompson lower bound opacity is attained at *just* low enough a temperature to support the required stellar heat transport. The second would be the good fortune that identical particles were involved in the first stellar reaction, thus suppressing the reaction rate due to the exclusion principle, and hence creating stars of sufficient longevity to support biological evolution. The third piece of luck, undeniable surely, would be that the strong nuclear force was just strong enough to bind the diproton – without which there would be no nuclear reactions and hence no stars and no chemical elements!

5. CONCLUSIONS

Increases of g_s sufficient to bind the diproton do not lead to significant production of diprotons in the immediate post-Big Bang period. This has been demonstrated for increases in g_s of up to 40%, corresponding to diprotons being roughly twice as stable as deuterons are in the actual universe (i.e. twice the binding energy).

The stability of diprotons would lead to a reaction pathway for converting protons to deuterons 10 or more orders of magnitude faster than the usual weak capture reaction under solar conditions. This would prevent stars of the familiar hot, dense type from occurring in the universe. Nevertheless, elementary stellar stability requirements can be met for cooler, lower density, conditions when the diproton is stable. These might give rise to stars with lifetimes of the order of billions of years, with luminosities and surface temperatures appropriate for the nurturing of biological life based on conventional molecular chemistry.

The above observations challenge the contention that the strong nuclear force has a fine-tuned anthropic upper bound requiring the diproton to be unbound.

However, this does not preclude there being other mechanisms which might impose an anthropic upper bound to the strong nuclear force. One possibility is the well-known 'Hoyle' resonance energies which promote the production of carbon and oxygen. We also note that increases in g_s rather greater than 40% would lead to deuterium being stable before 1 second. At such times the leptonic reactions which interconvert protons and neutrons were still active. The neutrons would thus escape into the sanctuary of helium-4 whilst the numbers of neutrons and protons were comparable. The universe would thus contain little hydrogen.

6. REFERENCES

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Appendix: Stellar "ppI" Reaction Sequence With A Stable Diproton

The possibilities for the reaction sequence, analogous to the usual ppI sequence, are listed below. Reactions involving nuclei with Z > 2, analogous to the ppII/ppIII sequences, have been ignored for simplicity, as have reactions involving neutrons as a reactant. The reaction rates given below have been taken from Ref.6. No correction has been made for the increased strength of the nuclear force. For the electromagnetic reactions, this is reasonable. The justification for the other reactions is that reaction [1a] will be found to be the rate determining step. Hence, faster subsequent reactions will not cause a major change to the scenario outlined below.

		Rate at 10^6 K $s^{-1}(\text{mole/cm}^3)^{-1}$
[1a]	$p + p \rightarrow {}^2_2\text{He} + \gamma$	6.6 x 10^{-14}
[1b]	${}^2_2\text{He} \rightarrow {}^2_1\text{D} + e^+ + \nu_e$	*Assumed fast*
[2]	$p + D \rightarrow {}^3_2\text{He} + \gamma$	3.4 x 10^{-12}
[3]	${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2{}^1_1p$	1.46 x 10^{-41}
[4]	${}^2_1\text{D} + {}^2_1\text{D} \rightarrow {}^4_2\text{He} + \gamma$	7.85 x 10^{-16}
[5]	${}^2_1\text{D} + {}^2_1\text{D} \rightarrow {}^3_1\text{H} + {}^1_1p$	6.72 x 10^{-9}
[6]	${}^2_1\text{D} + {}^2_1\text{D} \rightarrow {}^3_2\text{He} + {}^1_0n$	6.33 x 10^{-9}
[7]	${}^3_1\text{H} + {}^2_1\text{D} \rightarrow {}^4_2\text{He} + {}^1_0n$	1.88 x 10^{-7}
[8]	${}^3_1\text{H} + {}^1_1p \rightarrow {}^4_2\text{He} + \gamma$	*7.5 x 10^{-12*}*
[9]	${}^3_2\text{He} + {}^2_1\text{D} \rightarrow {}^4_2\text{He} + {}^1_1p$	3.84 x 10^{-19}
[10]	${}^3_2\text{He} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^2_1\text{D}$	1.42 x 10^{-22}
[11]	${}^3_2\text{He} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0n + {}^1_1p$	2.0 x 10^{-22}

**Extrapolated from Ref.5 data at 30 x 10^6 K.*

The usual ppI sequence involves reactions [2] and [3]. At the low temperature considered, the reactions involving reactant nuclei with double charges are strongly suppressed by the Coulomb barrier. In particular, reaction [3] is not active and so the ppI sequence in our alternative universe must follow a different path. Reactions [4], [9], [10] and [11] are also too slow to contribute significantly. The dominant reaction sequence is thus,

[1a]→[1b] → [2] + [6] (^{3}He production)
[1a]→[1b] → [5] → [7] + [8] (^{4}He production)

Note that helium-3 is not burnt at this temperature. The above rates are consistent with the timescale of the hydrogen burning phase being determined by reaction [1a]. Equilibrium deuteron and tritium densities are around 10^{-3} and 10^{-4} of the proton density respectively. The end product of the hydrogen burning phase is a mixture of roughly 50% helium-3 and 50% helium-4. Such a star would exhibit a distinct helium-3 burning phase after the hydrogen phase (following additional gravitational collapse to raise the temperature sufficiently to activate reaction [3]). Only then would the usual helium-4 burning phase occur.

The power density suggested in the main text is an upper bound based on full conversion to helium-4 in the hydrogen burning phase. The heat released per helium nucleus equals, to a good enough approximation, the helium-4 binding energy less twice the neutron/proton mass difference. The binding energy is increased significantly in our hypothetical universe. Based on the energy levels of a square potential well, we estimate the binding energy to be,

$$\frac{B}{B_{actual}} = \left(\frac{g_s / g_s^{actual} - 0.85}{1.0 - 0.85} \right)^2$$

because a reduction in g_s to $\sim 0.85 g_s^{actual}$ results in the deuteron being unbound (i.e. B = 0). Obviously this is a rather crude approximation, but is probably more indicative than, say, a factor of $\left(g_s / g_s^{actual}\right)^4$ which some authors have employed in analogy with the energy levels of atomic hydrogen. It results in helium-4 binding energies of 154, 255 and 380 MeV respectively, for g_s increased by x1.2, x1.3, x1.4. The energy release is thus quite prodigious by normal standards.